

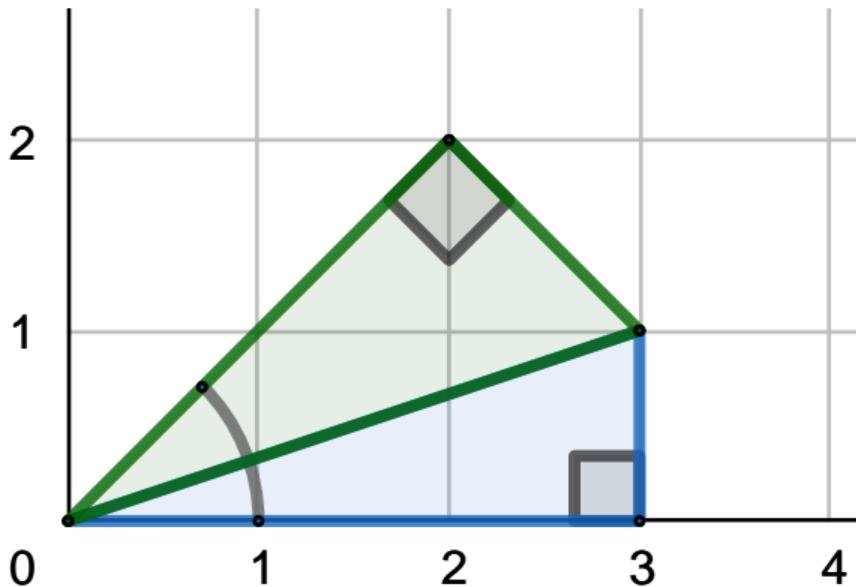
アークタンジェント公式

$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{3}$	Euler
$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239}$	Machin
$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{7}$	Hutton
$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{8}$	Strassnitzky
$\frac{\pi}{4} = 2 \arctan \frac{1}{2} - \arctan \frac{1}{7}$	Hermann
$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{4} + \arctan \frac{1}{13}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{12} + \arctan \frac{1}{17}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{8} + \arctan \frac{1}{57}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{9} + \arctan \frac{1}{32}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{2} - \arctan \frac{1}{5} + \arctan \frac{1}{18}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{2} - \arctan \frac{1}{9} - \arctan \frac{1}{32}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{2} - \arctan \frac{1}{6} + \arctan \frac{1}{43}$	Størmer
$\frac{\pi}{4} = 2 \arctan \frac{1}{2} - \arctan \frac{1}{8} - \arctan \frac{1}{57}$	Størmer
$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{13} + \arctan \frac{1}{21}$	Matsumoto
$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{4} + \arctan \frac{1}{23} + \arctan \frac{1}{30}$	Matsumoto
$\frac{\pi}{4} = \arctan \frac{1}{3} + \arctan \frac{1}{5} + \arctan \frac{1}{7} + \arctan \frac{1}{8}$	Takano
$\frac{\pi}{4} = \arctan \frac{1}{3} + \arctan \frac{1}{4} + \arctan \frac{1}{7} + \arctan \frac{1}{13}$	Takano
$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{9} + \arctan \frac{1}{73}$	Takano
$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{4} + \arctan \frac{1}{15} + \arctan \frac{1}{98}$	Takano
$\frac{\pi}{2} = 2 \arctan \frac{1}{2} + \arctan \frac{4}{7} + \arctan \frac{1}{8}$	Newton
$\frac{\pi}{4} = \arctan \frac{1}{7} + \arctan \frac{1}{8} + \arctan \frac{4}{7}$	

$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{3}$$

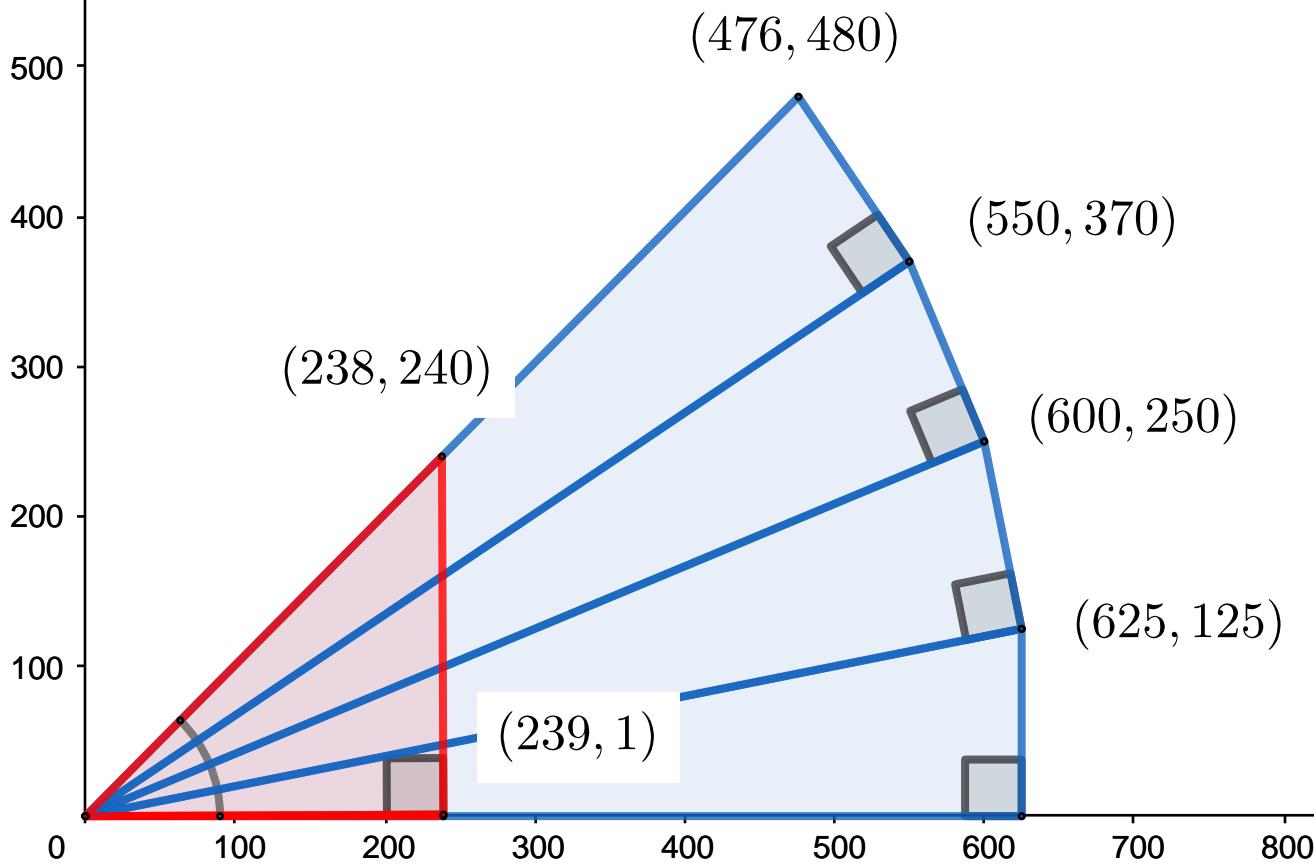
Euler

another solution of Nelsen (1993) page 39



$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239} \quad \text{Machin}$$

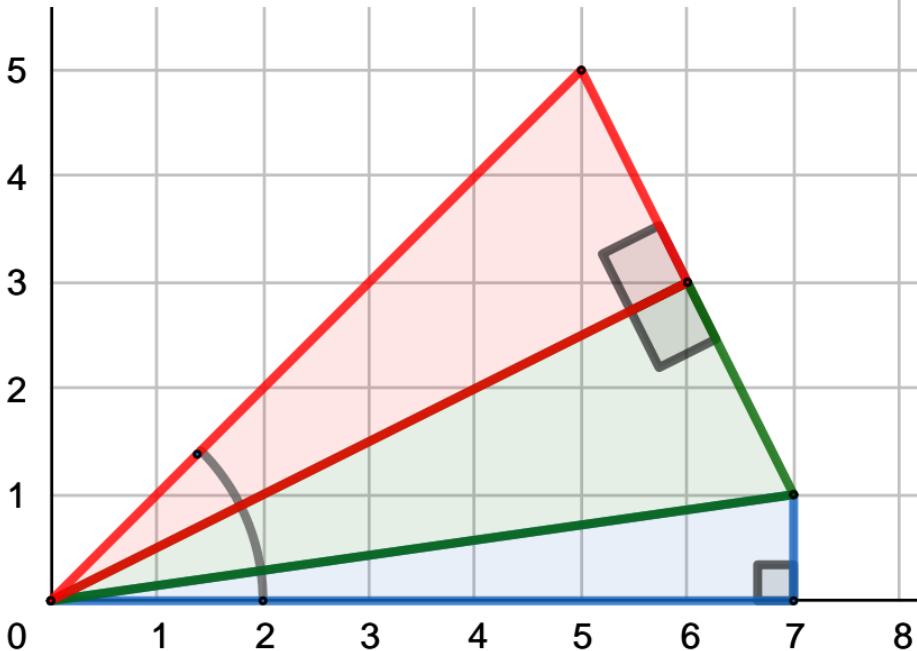
another solution of Heo (2015) and Nelsen (1990)



$$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{7}$$

Hutton

Reference: Nelsen (2015) page 75

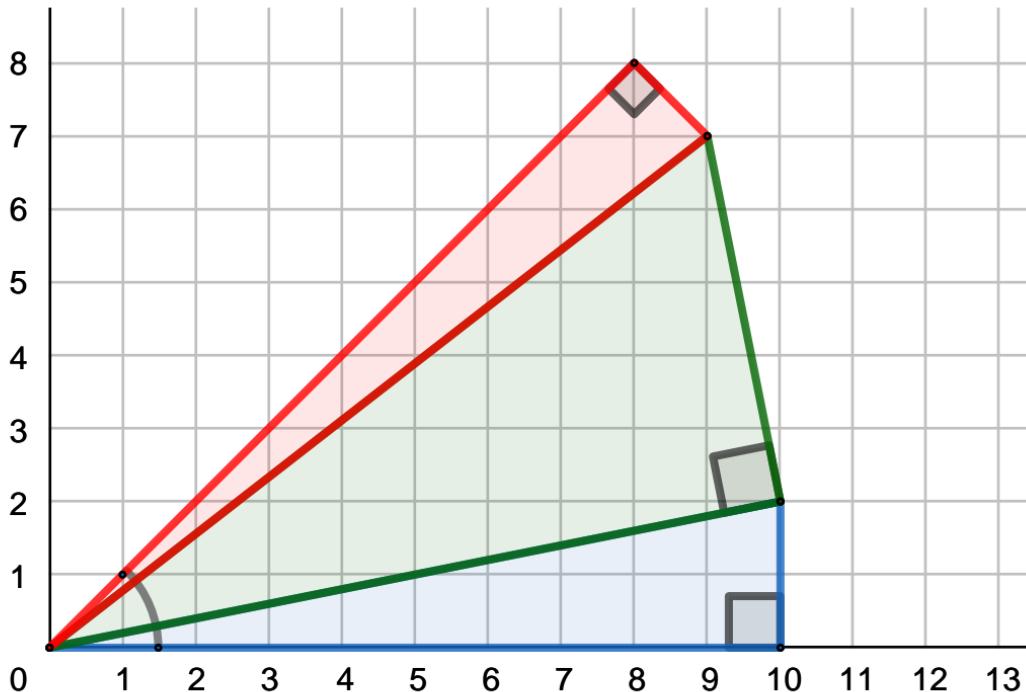


12

$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{8}$$

Strassnitzky

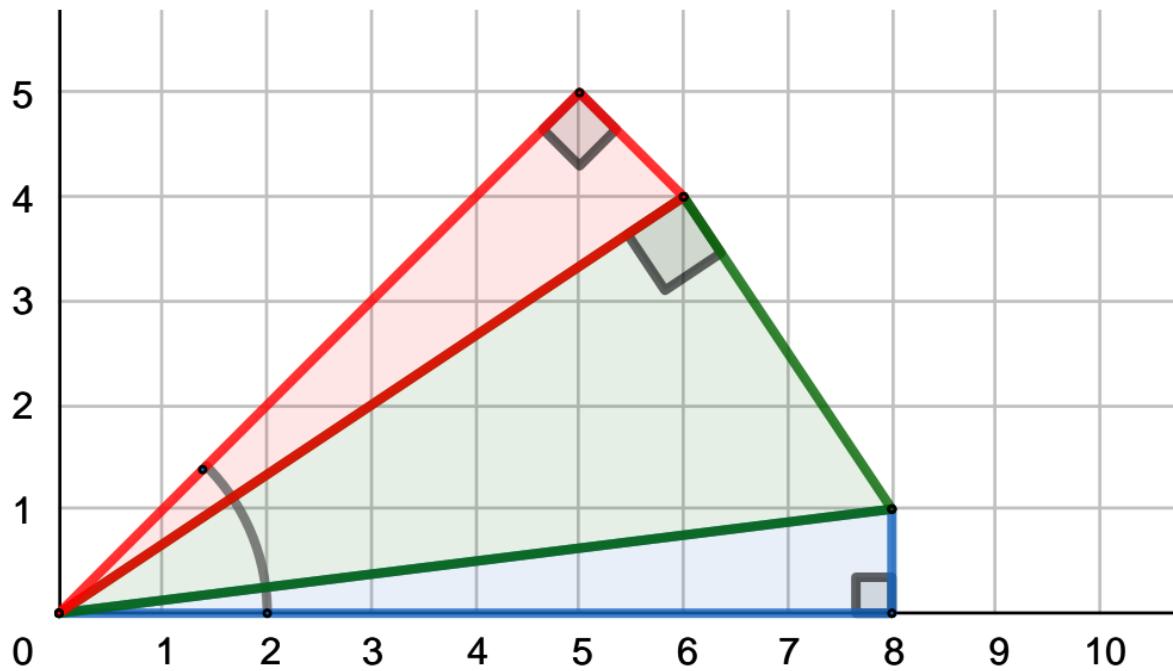
Reference: Nelsen (2015) page 75



$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{8}$$

Strassnitzky

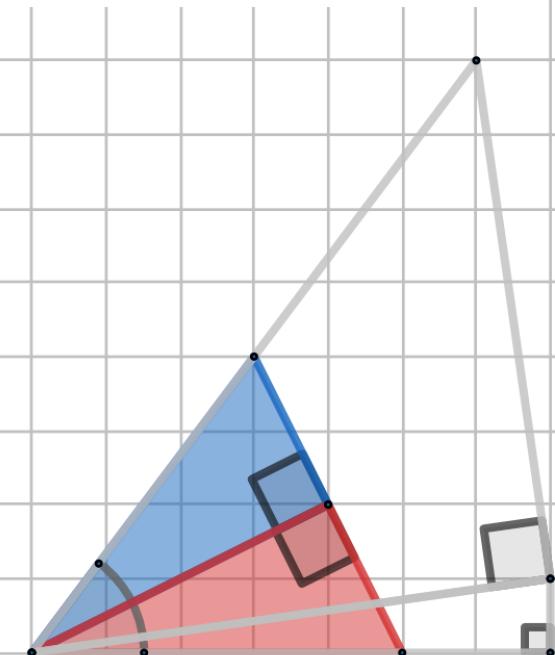
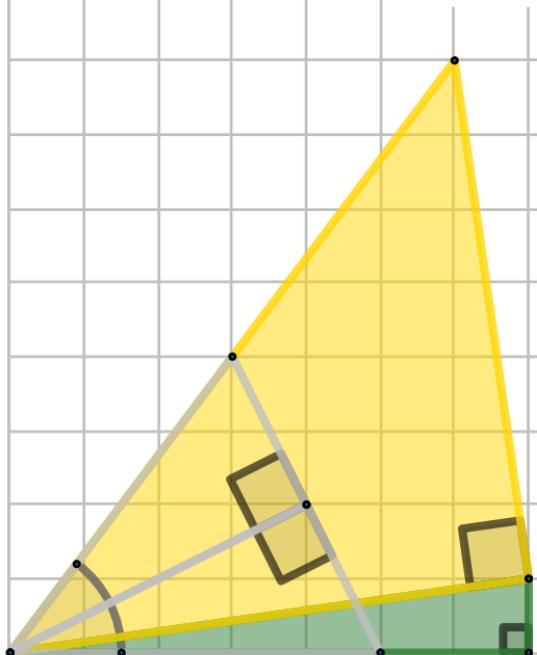
I improved Nelsen (2015) page 75.



$$\arctan \frac{1}{7} + \frac{\pi}{4} = 2 \arctan \frac{1}{2}$$

Hermann

I slightly improved Unal (2014a).



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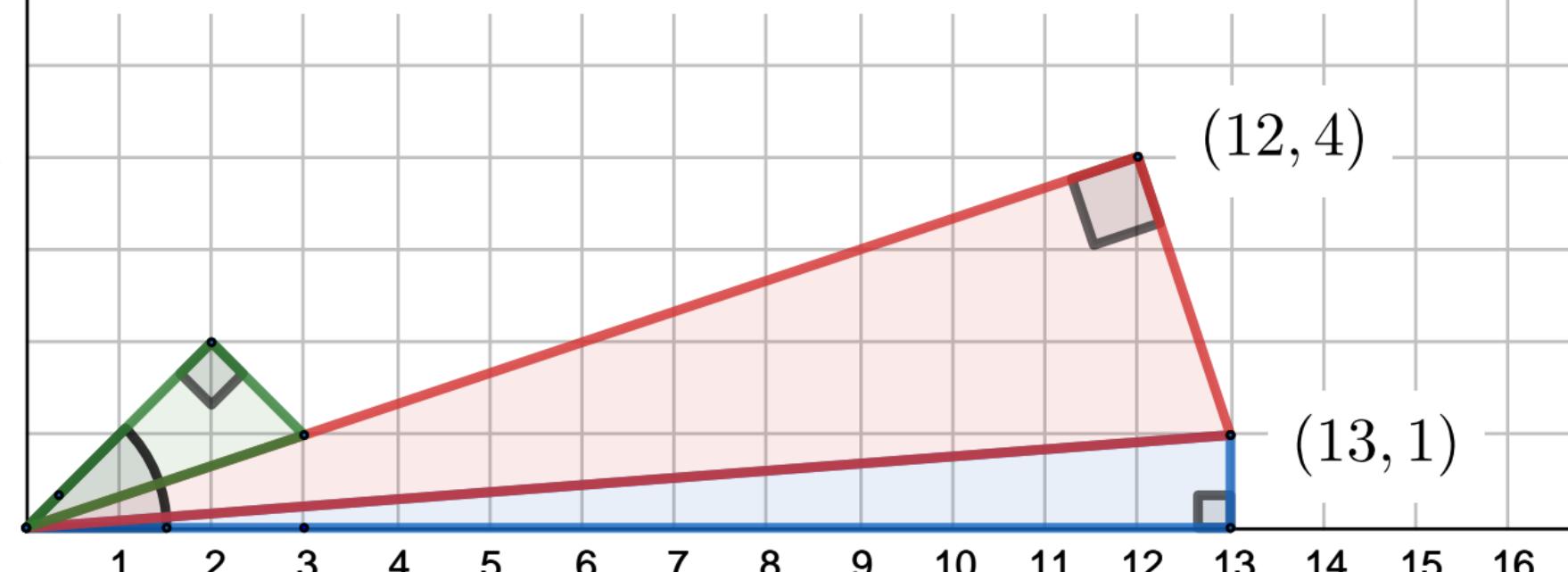
12

$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{4} + \arctan \frac{1}{13}$$

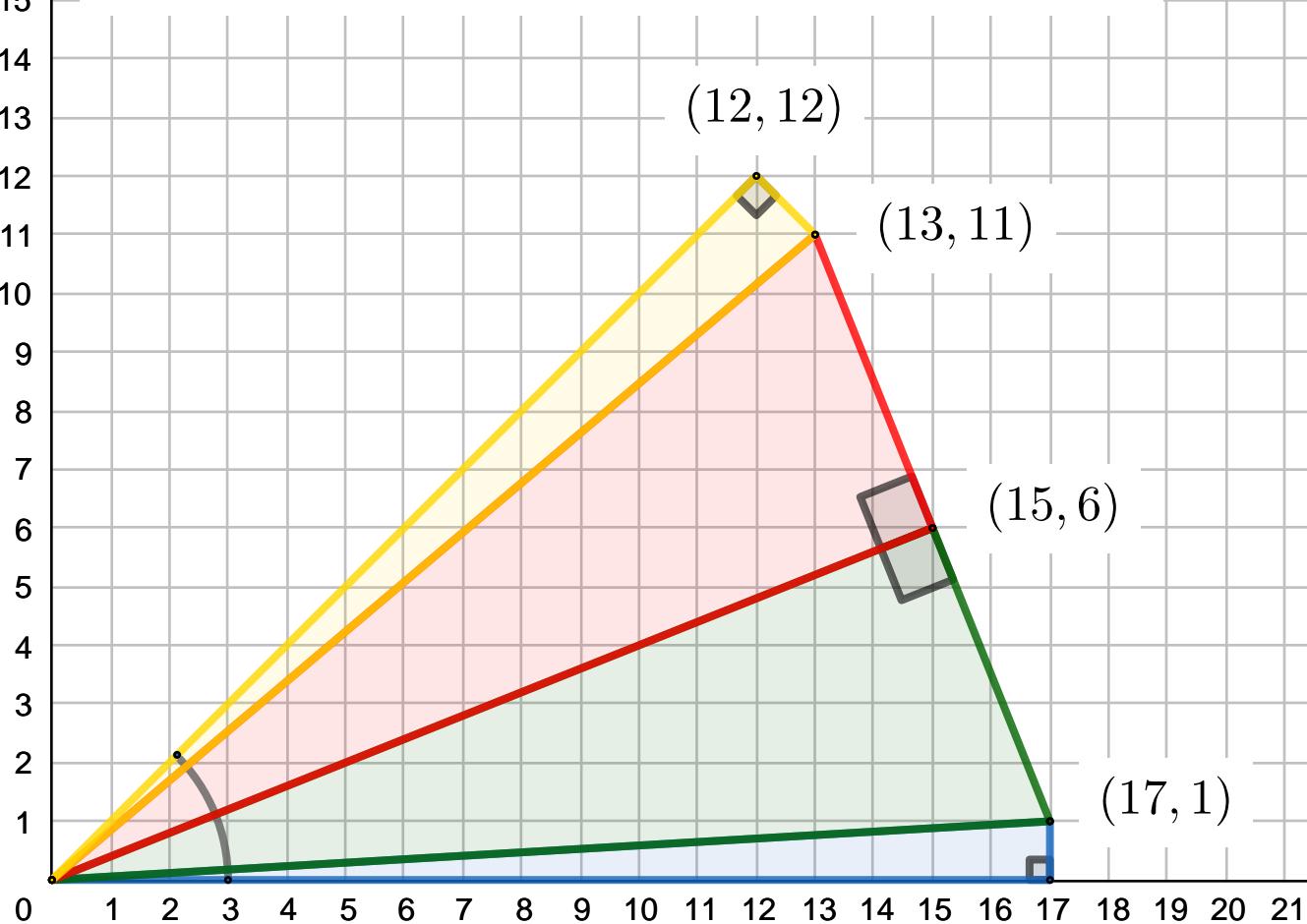
Stormer

(12, 4)

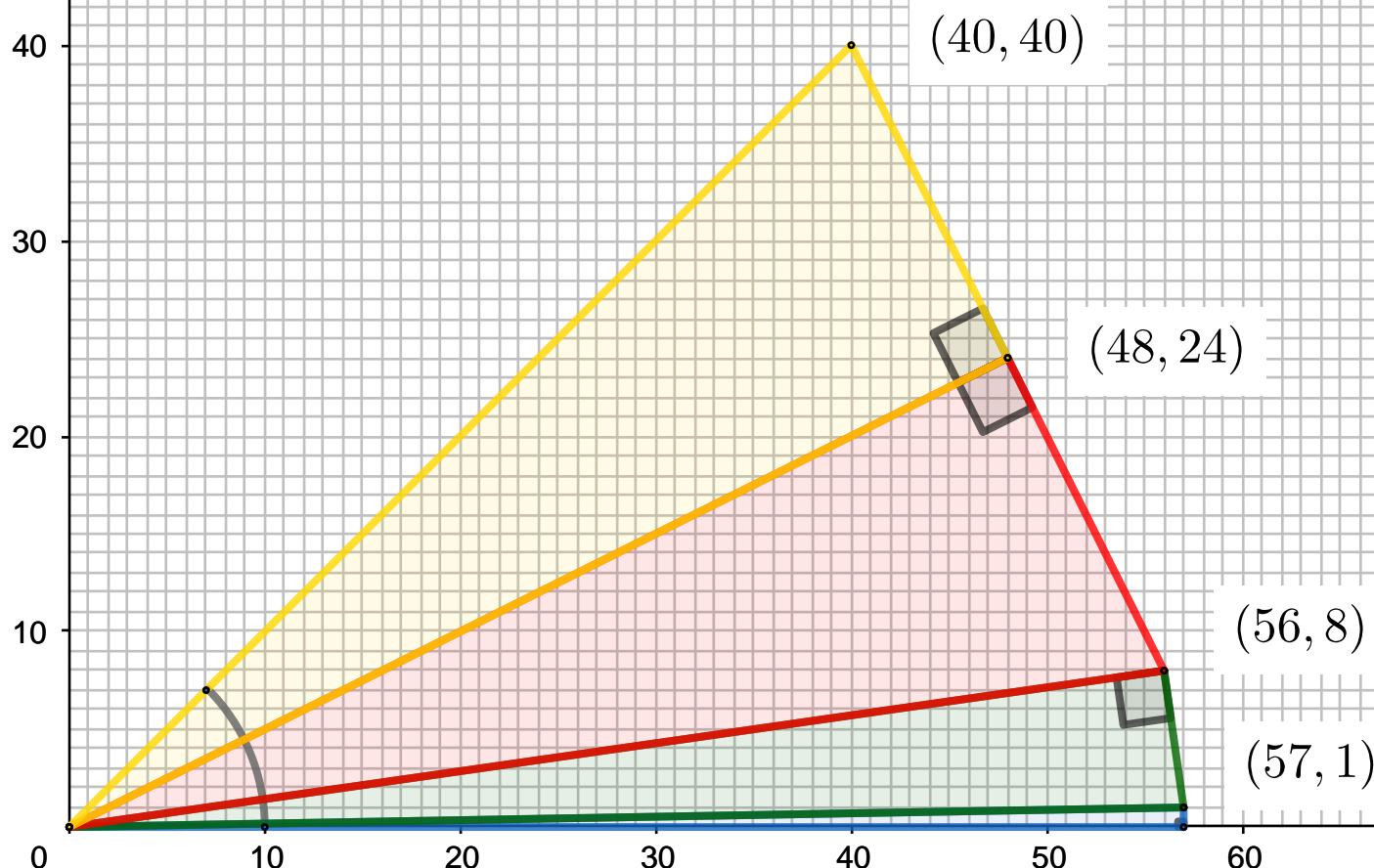
(13, 1)



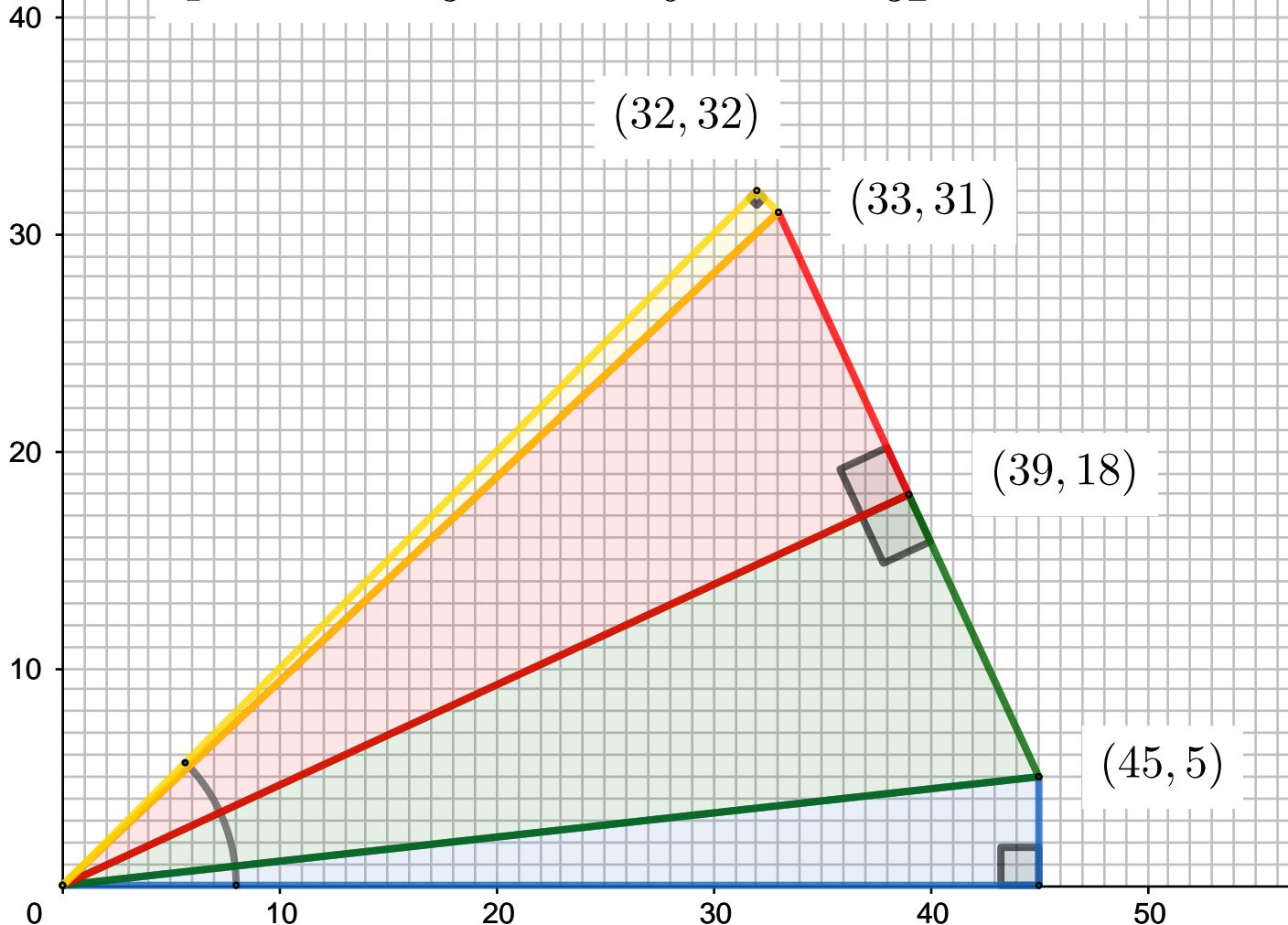
$$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{12} + \arctan \frac{1}{17} \quad \text{Stormer}$$



$$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{8} + \arctan \frac{1}{57} \quad \text{Stormer}$$

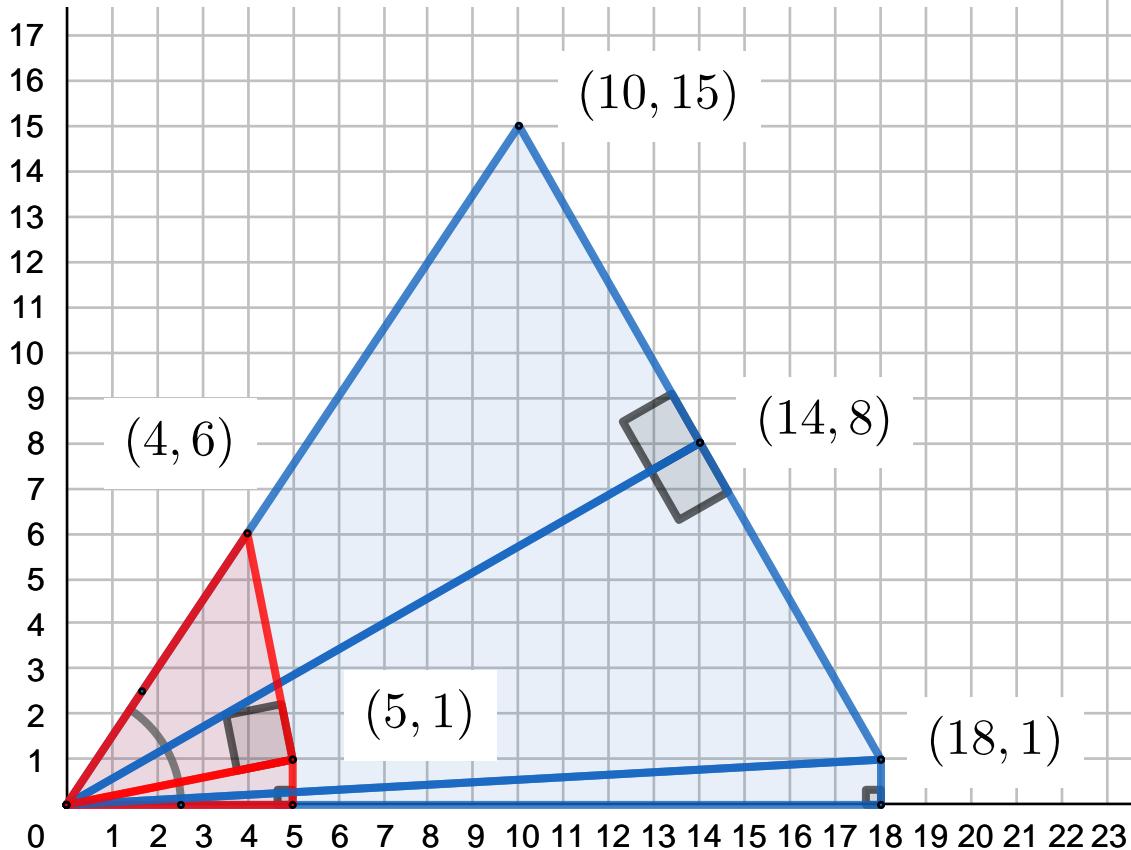


$$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{9} + \arctan \frac{1}{32} \quad \text{Stormer}$$



$$\arctan \frac{1}{5} + \frac{\pi}{4} = 2 \arctan \frac{1}{2} + \arctan \frac{1}{18}$$

Stormer



$$\arctan \frac{1}{32} + \frac{\pi}{4} + \arctan \frac{1}{9} = 2 \arctan \frac{1}{2} \quad \text{Stormer}$$

(27, 36)

(31, 33)

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(3, 4)

(4, 2)

(32, 1)

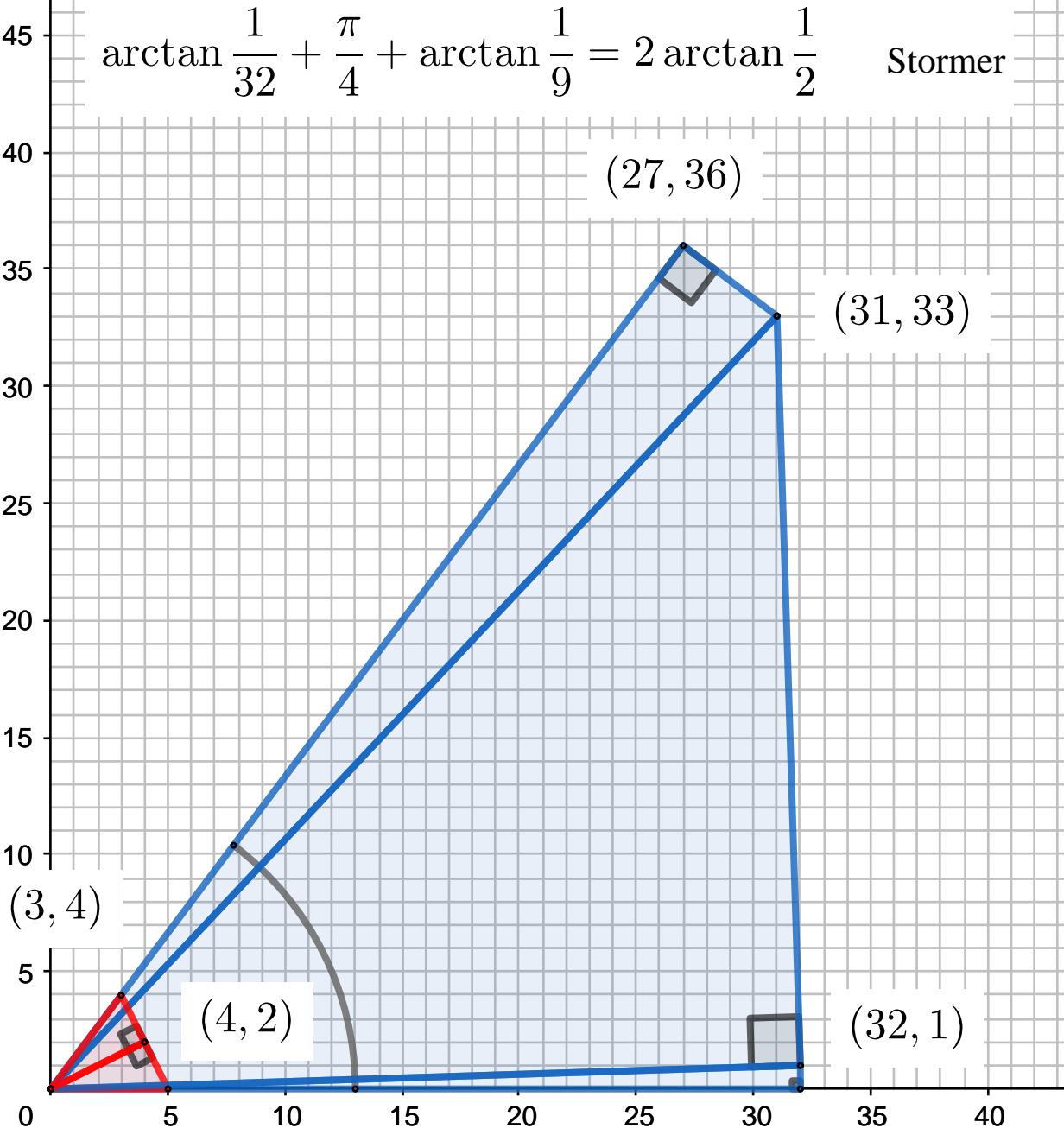
20

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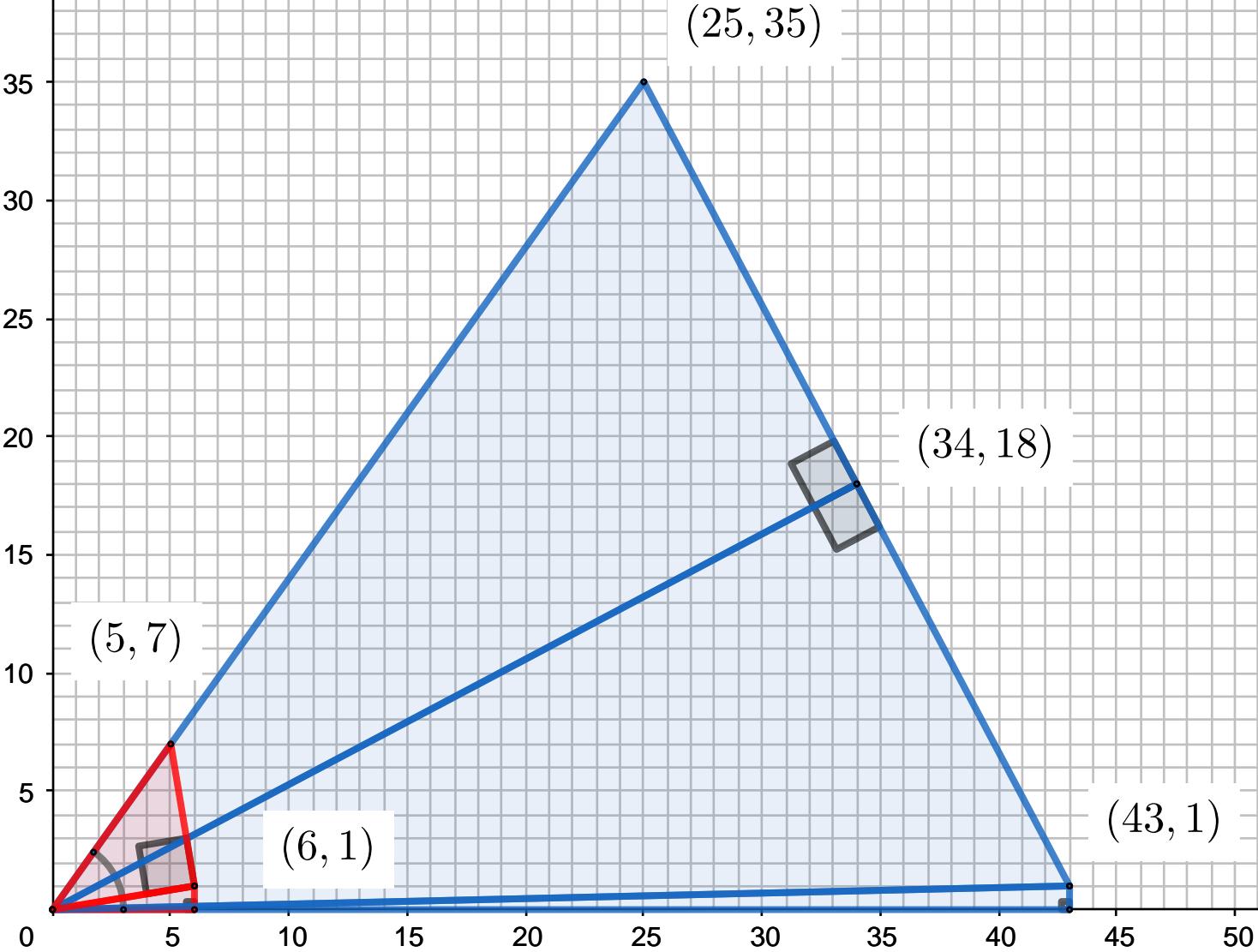
30

35

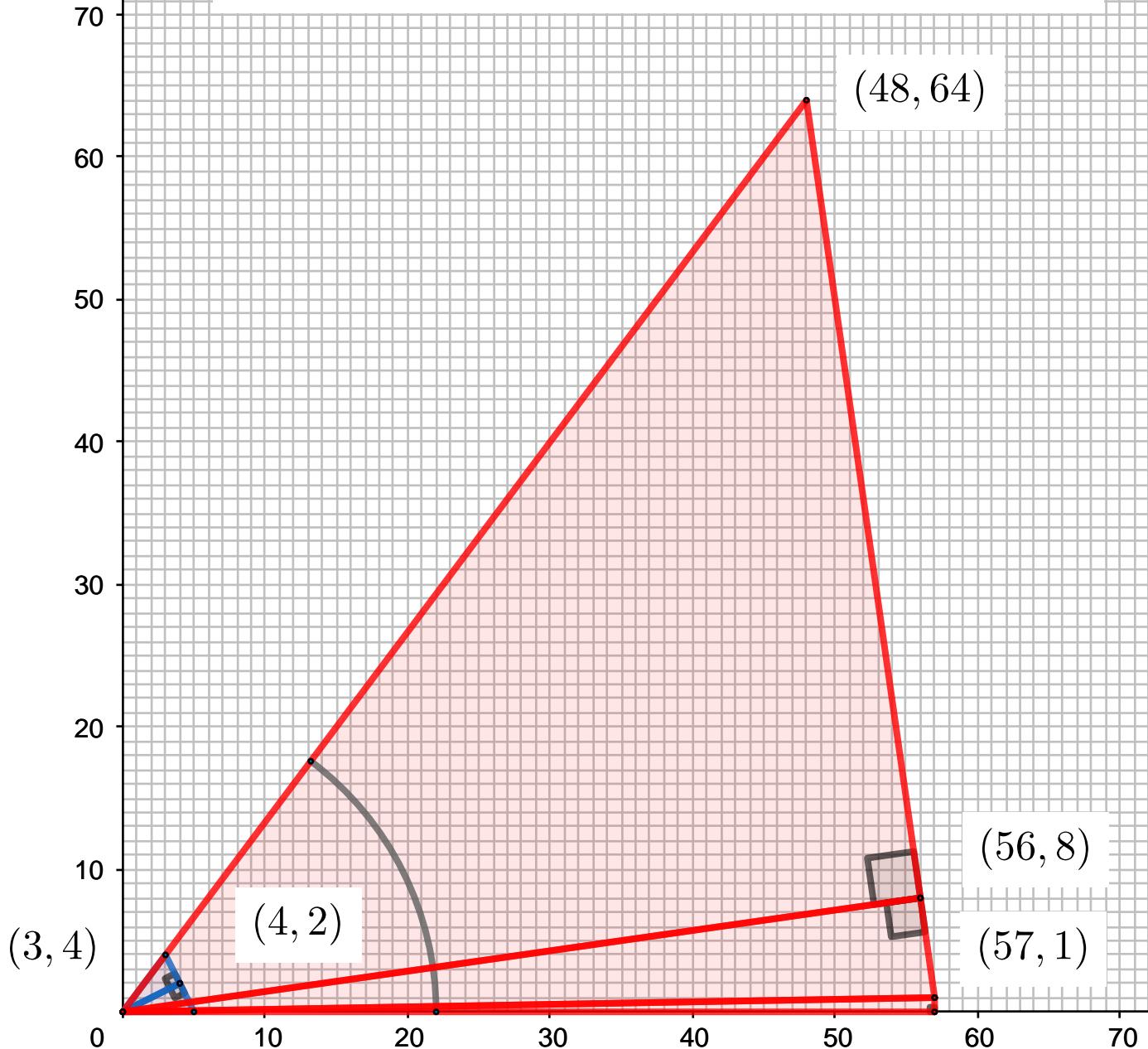
40



$$\arctan \frac{1}{6} + \frac{\pi}{4} = 2 \arctan \frac{1}{2} + \arctan \frac{1}{43} \quad \text{Stormer}$$

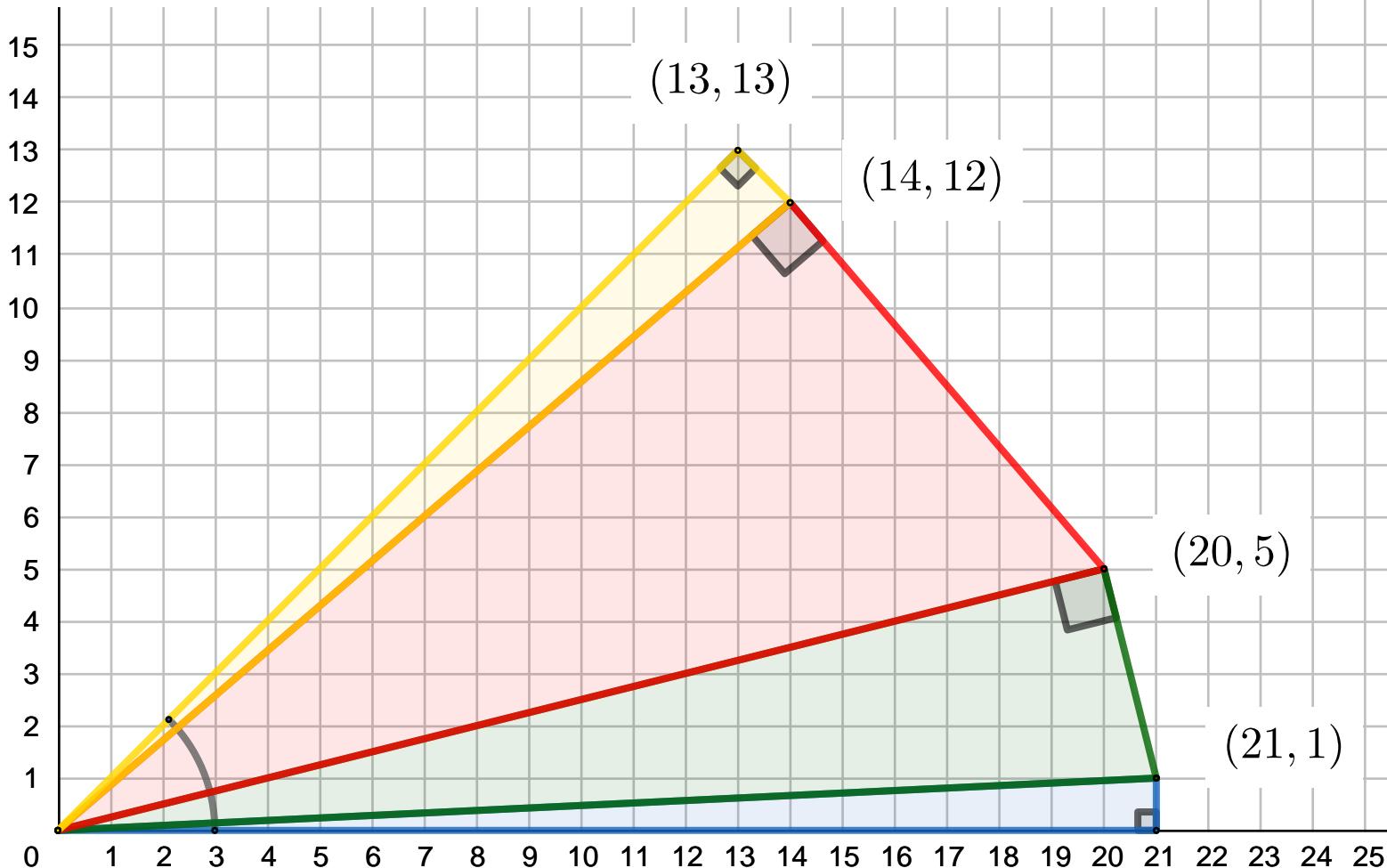


$$\arctan \frac{1}{57} + \arctan \frac{1}{8} + \frac{\pi}{4} = 2 \arctan \frac{1}{2} \quad \text{Stormer}$$

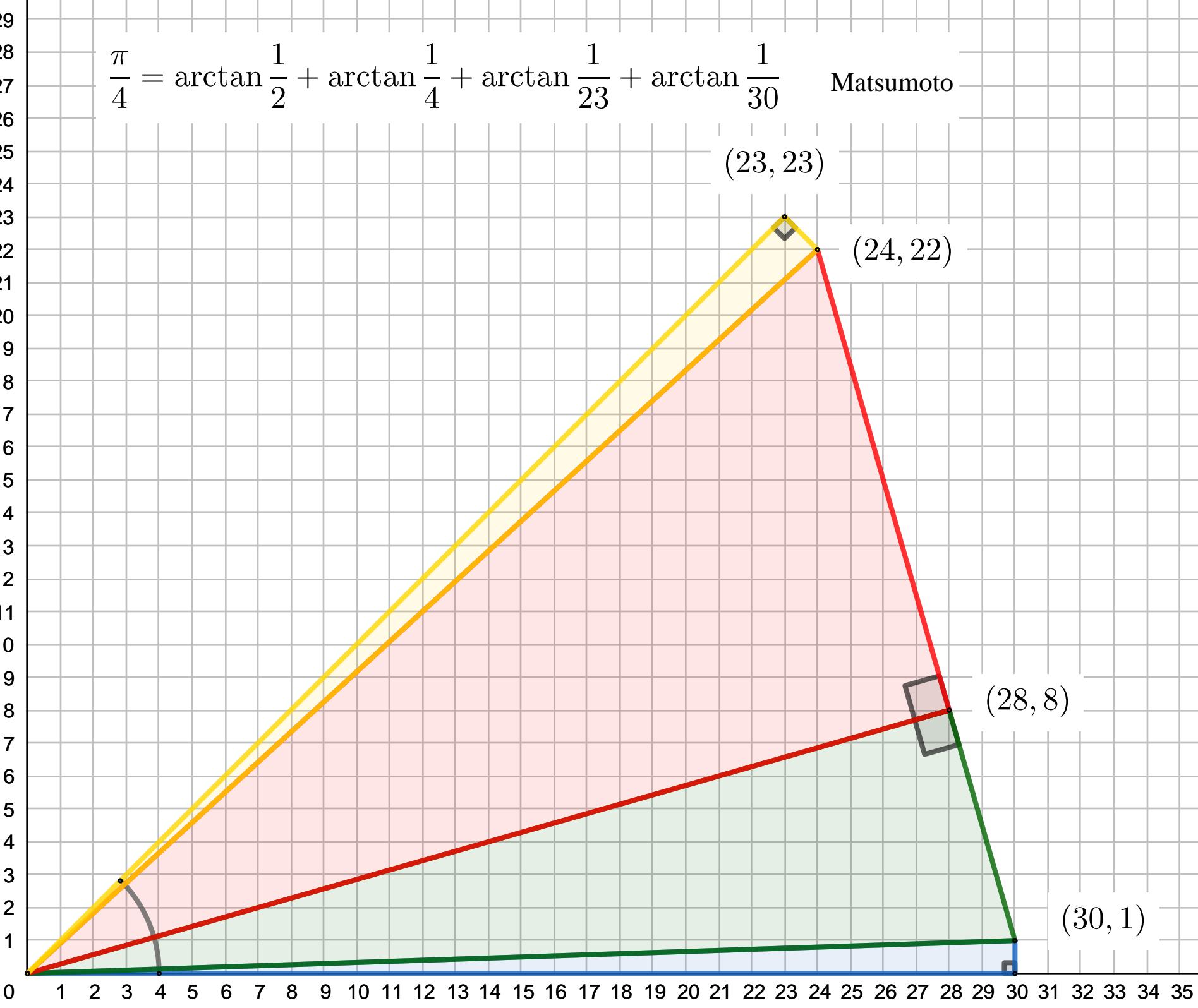


$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{13} + \arctan \frac{1}{21}$$

Matsumoto



$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{4} + \arctan \frac{1}{23} + \arctan \frac{1}{30} \quad \text{Matsumoto}$$



$$\frac{\pi}{4} = \arctan \frac{1}{3} + \arctan \frac{1}{5} + \arctan \frac{1}{7} + \arctan \frac{1}{8}$$

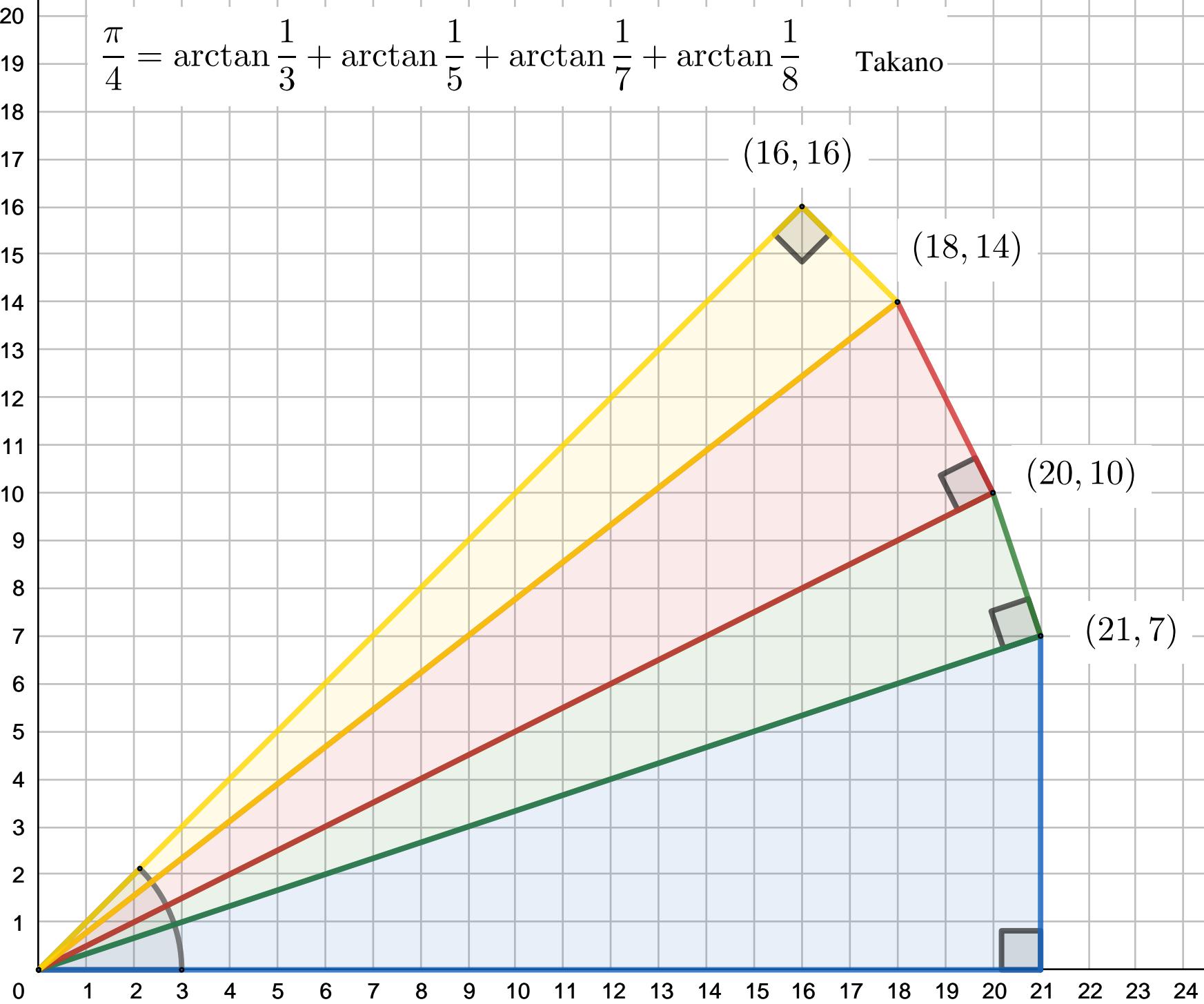
Takano

(16, 16)

(18, 14)

(20, 10)

(21, 7)



$$\frac{\pi}{4} = \arctan \frac{1}{3} + \arctan \frac{1}{4} + \arctan \frac{1}{7} + \arctan \frac{1}{13}$$

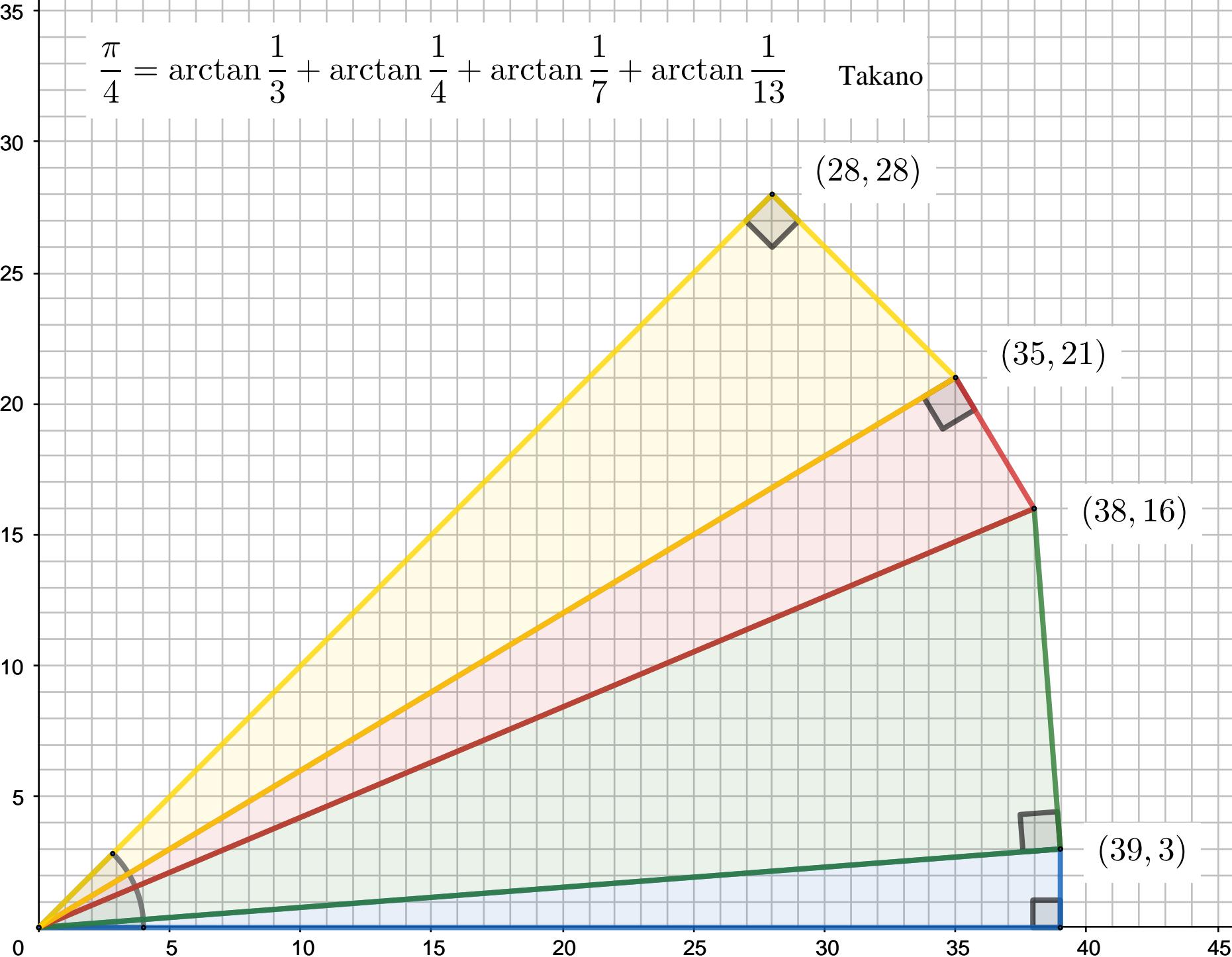
Takano

(28, 28)

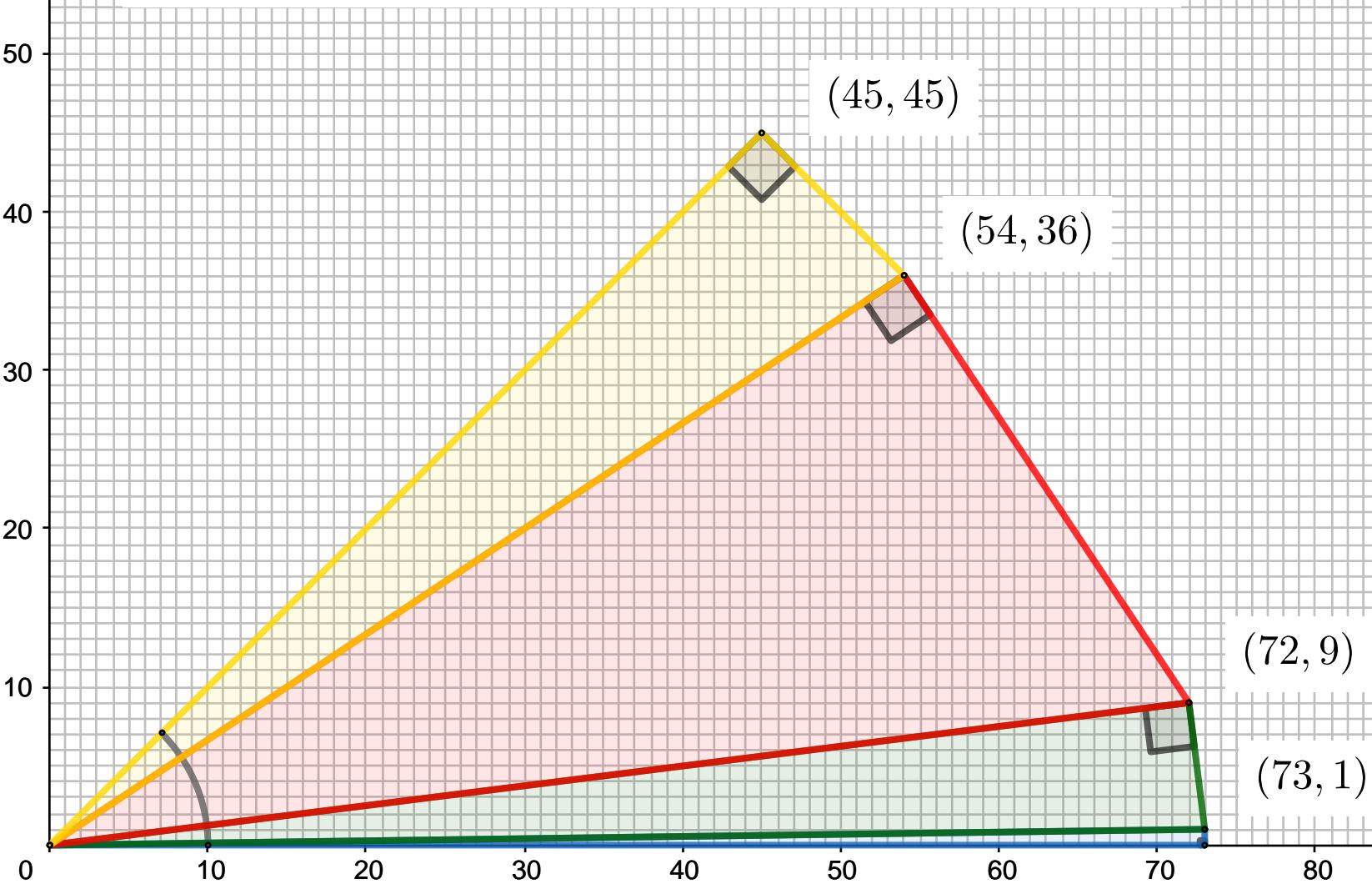
(35, 21)

(38, 16)

(39, 3)



$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{9} + \arctan \frac{1}{73} \quad \text{Takano}$$



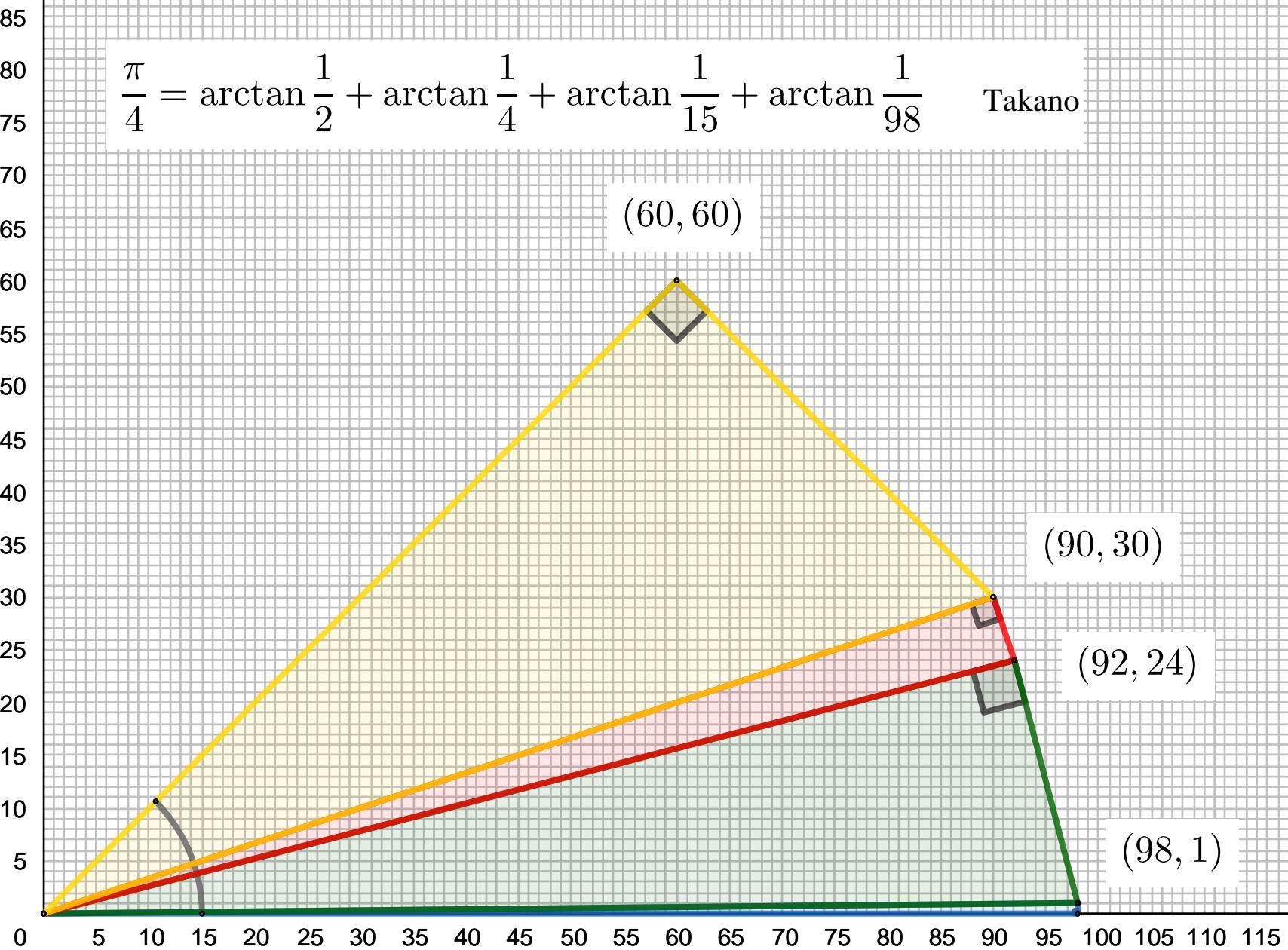
$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{4} + \arctan \frac{1}{15} + \arctan \frac{1}{98} \quad \text{Takano}$$

(60, 60)

(90, 30)

(92, 24)

(98, 1)



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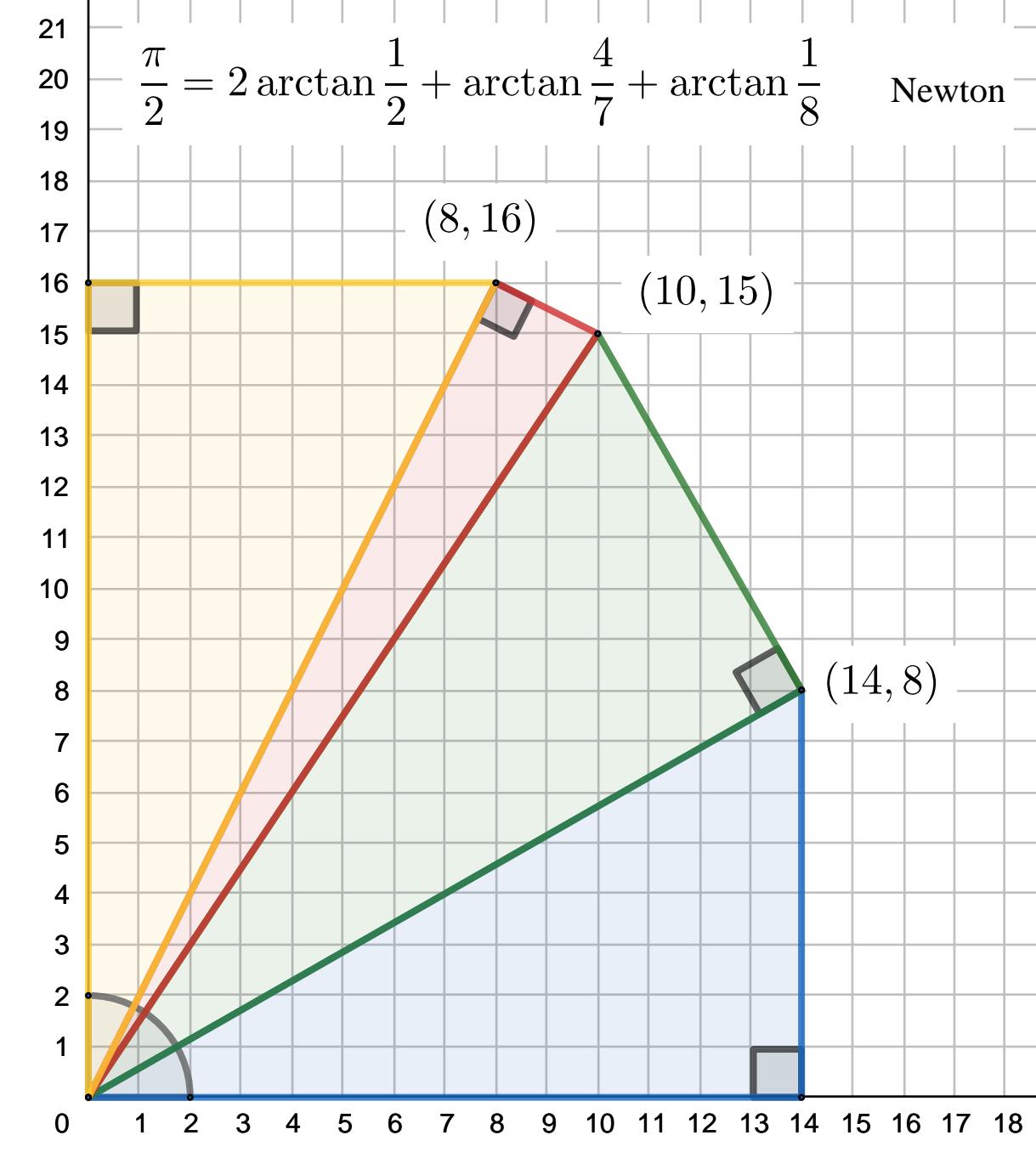
0

$$\frac{\pi}{2} = 2 \arctan \frac{1}{2} + \arctan \frac{4}{7} + \arctan \frac{1}{8}$$

Newton

(8, 16)

(10, 15)

(14, 8)

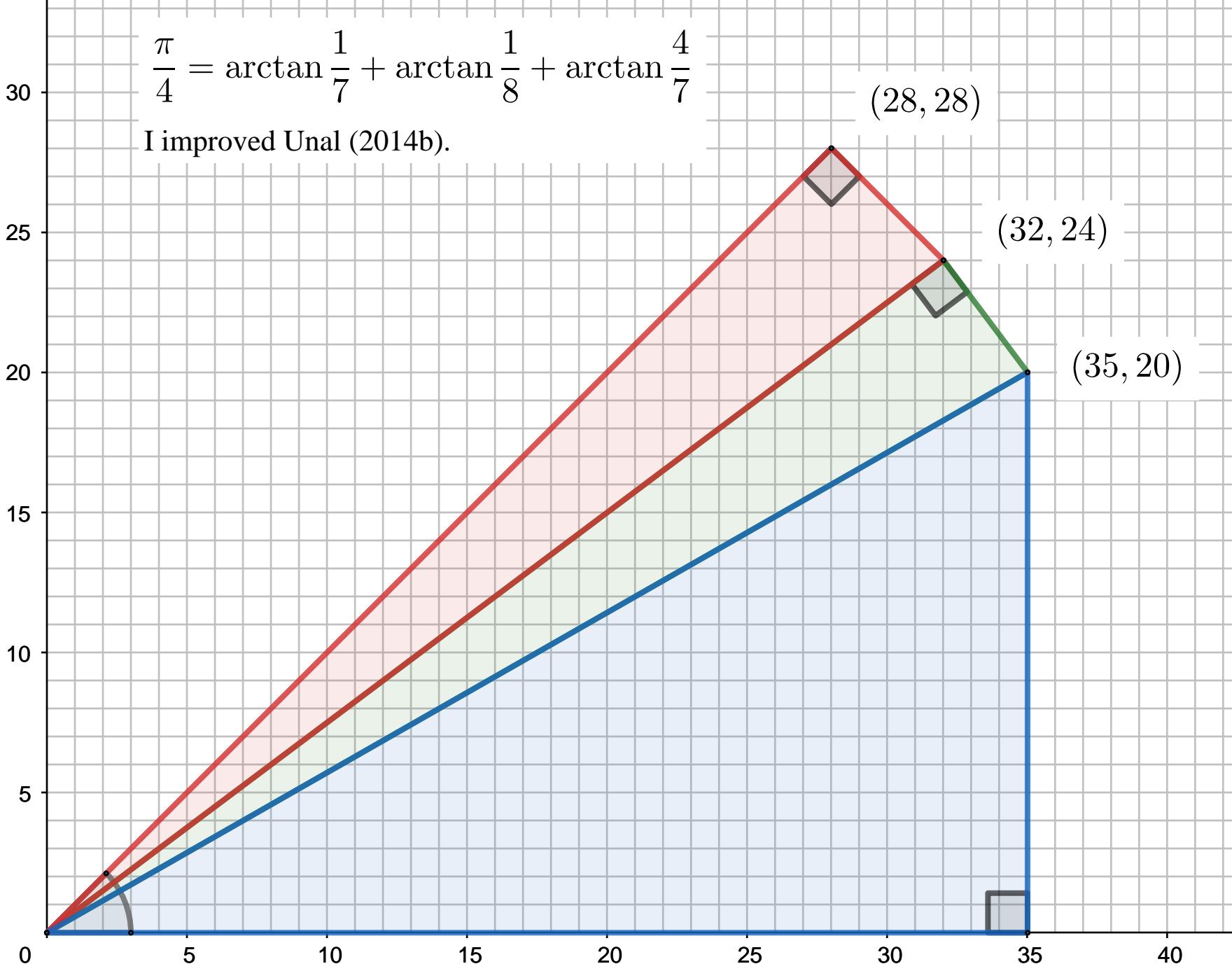
$$\frac{\pi}{4} = \arctan \frac{1}{7} + \arctan \frac{1}{8} + \arctan \frac{4}{7}$$

I improved Unal (2014b).

(28, 28)

(32, 24)

(35, 20)



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