

円周率カード

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$$\frac{1}{10} \left(47 - 9\sqrt{3} \right) < \pi < 6 - \frac{1}{4} - \frac{3}{320} - \frac{3}{2}\sqrt{3} \quad (1)$$

$$\frac{22}{7} - \frac{1}{630} < \pi < \frac{22}{7} - \frac{1}{1260} \quad (2)$$

$$\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \quad (3)$$

$$\frac{\pi}{8} = \sum_{n=0}^{\infty} \frac{1}{(4n+1)(4n+3)} \quad (4)$$

$$\pi = 3 + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n+1)^3 - (2n+1)} \quad (5)$$

数列 b_n を $b_1 = 0, b_2 = 1,$

$$b_{n+2} = \frac{1}{n}b_{n+1} + b_n, \quad n \geq 1$$

と定める. このとき,

$$\lim_{n \rightarrow \infty} \frac{n}{b_n^2} = \frac{\pi}{2}. \quad (6)$$

$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{3} \quad (7)$$

$$\frac{\pi}{4} = 2 \arctan \frac{1}{3} + \arctan \frac{1}{7} \quad (8)$$

$$\frac{\pi}{4} = \arctan \frac{1}{2} + \arctan \frac{1}{5} + \arctan \frac{1}{8} \quad (9)$$

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239} \quad (10)$$

$$\pi = 2\sqrt{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n} \quad (11)$$

$$\pi = 3 \sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2 (2n+1) 2^{4n}} \quad (12)$$

$$\frac{\pi}{2} = \prod_{n=1}^{\infty} \frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \quad (13)$$

$$\pi = \sum_{n=0}^{\infty} \frac{1}{2^{4n}} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \quad (14)$$

$$\pi = 16 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^5 + 4(2n+1)} \quad (15)$$

$$\frac{\pi}{4} = 44 \arctan \frac{1}{57} + 7 \arctan \frac{1}{239} - 12 \arctan \frac{1}{682} + 24 \arctan \frac{1}{12943} \quad (16)$$

$$\frac{\pi}{4} = 12 \arctan \frac{1}{18} + 8 \arctan \frac{1}{57} - 5 \arctan \frac{1}{239} \quad (17)$$

$$\pi = 4 - 8 \sum_{n=1}^{\infty} \frac{1}{(4n)^2 - 1} \quad (18)$$

$$\pi = \frac{3\sqrt{3}}{4} - 6 \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{(2n+3)(2n-1)16^n} \quad (19)$$

$$\pi = \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} \left(\frac{2}{4n+1} + \frac{2}{4n+2} + \frac{1}{4n+3} \right) \quad (20)$$

数列 a_n 及び b_n を $a_0 = 1, b_0 = 0,$

$$a_n = \frac{4n}{2n-1}a_{n-1} - \frac{1}{2n-1}, \quad n \geq 1$$

$$b_n = \frac{4n}{2n-1}b_{n-1} + \frac{8n}{(2n-1)^2}, \quad n \geq 1$$

と定める. このとき,

$$\lim_{n \rightarrow \infty} \frac{b_n}{a_n} = \pi. \tag{21}$$

$$\pi = \sum_{n=0}^{\infty} \frac{50n - 6}{\binom{3n}{n} 2^n} \quad (22)$$

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \binom{2n}{n}^3 \frac{42n + 5}{2^{12n+4}} \quad (23)$$

$$\frac{\pi}{2} = \sum_{n=1}^{\infty} \frac{2^n}{n \binom{2n}{n}} \quad (24)$$

$$\pi = 3\sqrt{3} \sum_{n=1}^{\infty} \frac{1}{n \binom{2n}{n}} \quad (25)$$

$$\pi = -4 + \sum_{n=0}^{\infty} \frac{2^{n+1}}{\binom{2n}{n}} \quad (26)$$

数列 p_n , P_n 及び a_n を $p_0 = 3$, $P_0 = 2\sqrt{3}$,

$$P_{n+1} = \frac{2p_n P_n}{p_n + P_n}, \quad n \geq 0$$

$$p_{n+1} = \sqrt{p_n P_{n+1}}, \quad n \geq 0$$

$$a_n = \frac{1}{3}(2p_n + P_n), \quad n \geq 0$$

と定める.

このとき, 数列 p_n , P_n 及び a_n は円周率 π に収束する. (27)

数列 a_n, b_n, c_n, s_n 及び p_n を $a_0 = 1, b_0 = \frac{1}{\sqrt{2}}, s_0 = \frac{1}{2},$

$$\begin{aligned} a_n &= \frac{a_{n-1} + b_{n-1}}{2}, & n \geq 1 & & b_n &= \sqrt{a_{n-1}b_{n-1}}, & n \geq 1 \\ c_n &= a_n^2 - b_n^2, & n \geq 1 & & s_n &= s_{n-1} - 2^n c_n, & n \geq 1 \\ p_n &= \frac{2a_n^2}{s_n}, & n \geq 1 & & & & \end{aligned}$$

と定める.

このとき, 数列 p_n は円周率 π に収束する. (28)

数列 k_n 及び e_n を $k_0 = 3 - 2\sqrt{2}$, $e_0 = 6 - 4\sqrt{2}$,

$$k_n = \frac{1 - \sqrt{1 - k_{n-1}^2}}{1 + \sqrt{1 - k_{n-1}^2}}, \quad n \geq 1$$

$$e_n = e_{n-1}(1 + k_n)^2 - 2^{n+1}k_n, \quad n \geq 1$$

と定める.

このとき, 数列 e_n は $\frac{1}{\pi}$ に収束する. (29)

数列 a_n , r_n 及び s_n を $a_0 = \frac{1}{3}$, $s_0 = \frac{\sqrt{3}-1}{2}$,

$$r_n = \frac{3}{1 + 2(1 - s_{n-1}^3)^{\frac{1}{3}}}, \quad n \geq 1$$

$$s_n = \frac{r_n - 1}{2}, \quad n \geq 1$$

$$a_n = r_n^2 a_{n-1} - 3^{n-1} (r_n^2 - 1), \quad n \geq 1$$

と定める.

このとき, 数列 a_n は $\frac{1}{\pi}$ に収束する.

(30)

数列 y_n 及び z_n を $y_0 = \sqrt{2} - 1$, $z_0 = 6 - 4\sqrt{2}$,

$$y_n = \frac{1 - \sqrt[4]{1 - y_{n-1}^4}}{1 + \sqrt[4]{1 - y_{n-1}^4}}, \quad n \geq 1$$

$$z_n = z_{n-1}(1 + y_n)^4 - 2 \cdot 4^n y_n(1 + y_n + y_n^2), \quad n \geq 1$$

と定める.

このとき, 数列 z_n は $\frac{1}{\pi}$ に収束する. (31)

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)! 1103 + 26390n}{(n!)^4 396^{4n}} \quad (32)$$

$$\frac{1}{\pi} = \frac{1}{3528} \sum_{n=0}^{\infty} (-1)^n \frac{(4n)!}{(n!)^4 4^{4n}} \frac{1123 + 21460n}{882^{2n}} \quad (33)$$

$$\pi = 16\sqrt{3} \sum_{n=1}^{\infty} \frac{n}{(4n-3)(4n-1)3^{2n-1}} \quad (34)$$

円周率 π は無理数である. (35)

連分数展開

$$\frac{4}{\pi} = 1 + \frac{1^2}{2} + \frac{3^2}{2} + \frac{5^2}{2} + \frac{7^2}{2} + \frac{9^2}{2} + \frac{11^2}{2} + \frac{13^2}{2} + \dots \quad (36)$$

連分数展開

$$\frac{\pi}{2} = 1 + \frac{1}{1 + \frac{1 \cdot 2}{1 + \frac{2 \cdot 3}{1 + \frac{3 \cdot 4}{1 + \frac{4 \cdot 5}{1 + \frac{5 \cdot 6}{1 + \frac{6 \cdot 7}{1 + \frac{7 \cdot 8}{1 + \dots}}}}}}}} \quad (37)$$

連分数展開

$$\pi = 3 + \frac{1^2}{6 + \frac{3^2}{6 + \frac{5^2}{6 + \frac{7^2}{6 + \frac{9^2}{6 + \frac{11^2}{6 + \frac{13^2}{6 + \dots}}}}}} \quad (38)$$

連分数展開

$$\frac{4}{\pi} = 1 + \frac{1^2}{3} + \frac{2^2}{5} + \frac{3^2}{7} + \frac{4^2}{9} + \frac{5^2}{11} + \frac{6^2}{13} + \frac{7^2}{15} + \dots \quad (39)$$

連分数展開

$$\frac{\pi}{2} = 1 + \frac{2}{3 + \frac{1 \cdot 3}{4 + \frac{3 \cdot 5}{4 + \frac{5 \cdot 7}{4 + \frac{7 \cdot 9}{4 + \frac{9 \cdot 11}{4 + \frac{11 \cdot 13}{4 + \frac{13 \cdot 15}{4 + \dots}}}}}}}} \quad (40)$$

連分数展開

$$\frac{4}{\pi} = 1 + \frac{2}{7 + \frac{1 \cdot 3}{8 + \frac{3 \cdot 5}{8 + \frac{5 \cdot 7}{8 + \frac{7 \cdot 9}{8 + \frac{9 \cdot 11}{8 + \frac{11 \cdot 13}{8 + \frac{13 \cdot 15}{8 + \dots}}}}}}}} \quad (41)$$

連分数展開

$$\frac{8}{-8 + 3\pi} = 5 + \frac{1 \cdot 5}{7} + \frac{2 \cdot 6}{9} + \frac{3 \cdot 7}{11} + \frac{4 \cdot 8}{13} + \frac{5 \cdot 9}{15} + \frac{6 \cdot 10}{17} + \frac{7 \cdot 11}{19} + \dots \quad (42)$$

連分数展開

$$\frac{4}{-2 + \pi} = 3 + \frac{1 \cdot 3}{5} + \frac{2 \cdot 4}{7} + \frac{3 \cdot 5}{9} + \frac{4 \cdot 6}{11} + \frac{5 \cdot 7}{13} + \frac{6 \cdot 8}{15} + \frac{7 \cdot 9}{17} + \dots \quad (43)$$

$$\pi = 4 + \sum_{n=1}^{\infty} (-16)^n \frac{\binom{2n}{n} (40n^2 + 16n + 1)}{\binom{4n}{2n}^2 2n (4n + 1)^2} \quad (44)$$

$$\frac{1}{\pi} = 12 \cdot \sum_{n=0}^{\infty} \frac{(-1)^n (6n)!}{(3n)! (n!)^3} \cdot \frac{13591409 + 545140134n}{640320^{3n+3/2}} \quad (45)$$