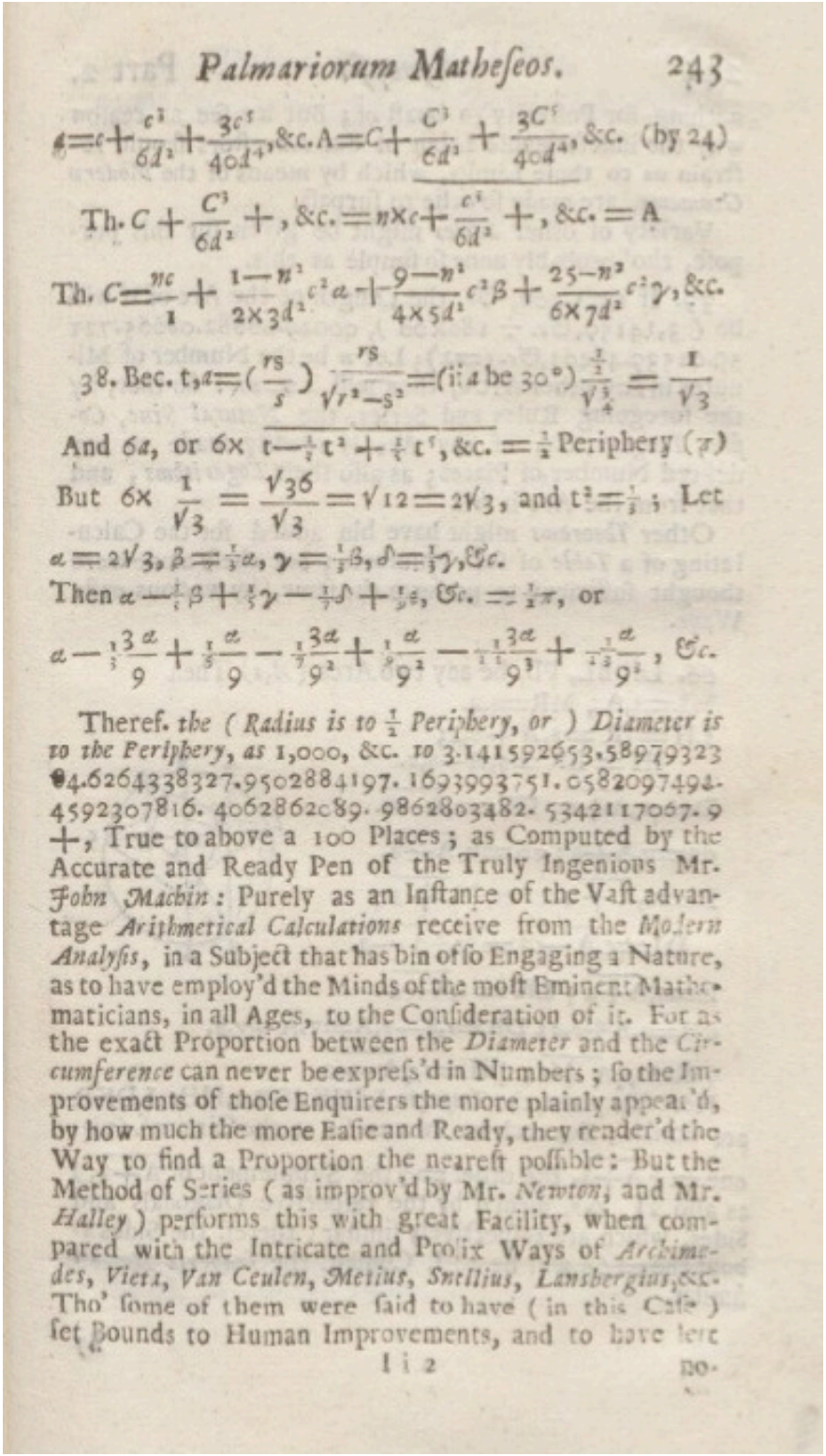
マチンらによる円周率πの数値計算

2025 年 12 月 6 日(土)

はじめに

1706 年マチンは円周率πを 100 桁まで計算し世界記録を樹立しました. その証拠として (Jones, 1706)　の 243 ページに 100 桁の数字が記載されています. 彼は如何なる方法で円周率πを計算したのでしょうか?



Palmariorum Matheſeos. 243

$$A = c + \frac{c^3}{6d^2} + \frac{3c^5}{40d^4}, \&c.\ A = C + \frac{C^3}{6d^2} + \frac{3C^5}{40d^4}, \&c. \text{ (by 24)}$$

$$\text{Th. } C + \frac{C^3}{6d^2} +, \&c. = n \times c + \frac{c^3}{6d^2} +, \&c. = A$$

$$\text{Th. } C = \frac{nc}{1} + \frac{1-n^2}{2\times 3d^2}c^2\alpha + \frac{9-n^2}{4\times 5d^2}c^2\beta + \frac{25-n^2}{6\times 7d^2}c^2\gamma, \&c.$$

$$38.\ \text{Bec. } t, a = \left(\frac{rs}{s}\right)\frac{rs}{\sqrt{r^2-s^2}} = (\text{if } a \text{ be } 30^\circ)\frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} = \frac{1}{\sqrt{3}}$$

And $6a$, or $6\times \overline{t - \frac{1}{3}t^3 + \frac{1}{5}t^5, \&c.} = \frac{1}{2}$ Periphery (π)

But $6\times \frac{1}{\sqrt{3}} = \frac{\sqrt{36}}{\sqrt{3}} = \sqrt{12} = 2\sqrt{3}$, and $t^2 = \frac{1}{3}$; Let

$\alpha = 2\sqrt{3}, \beta = \frac{1}{3}\alpha, \gamma = \frac{1}{3}\beta, \delta = \frac{1}{3}\gamma, \&c.$

Then $\alpha - \frac{1}{3}\beta + \frac{1}{5}\gamma - \frac{1}{7}\delta + \frac{1}{9}\varepsilon, \&c. = \frac{1}{2}\pi$, or

$$\alpha - \frac{1}{3}\frac{3\alpha}{9} + \frac{1}{5}\frac{\alpha}{9} - \frac{1}{7}\frac{3\alpha}{9^2} + \frac{1}{9}\frac{\alpha}{9^2} - \frac{1}{11}\frac{3\alpha}{9^3} + \frac{1}{13}\frac{\alpha}{9^3}, \&c.$$

Theref. *the (Radius is to* $\frac{1}{2}$ *Periphery, or) Diameter is to the Periphery, as* 1,000, &c. *to* 3.141592653.58979323 84.6264338327.9502884197. 1693993751. 0582097494. 4592307816. 4062862089. 9862803482. 5342117067. 9 +, True to above a 100 Places; as Computed by the Accurate and Ready Pen of the Truly Ingenious Mr. *John Machin:* Purely as an Inſtance of the Vaſt advantage *Arithmetical Calculations* receive from the *Modern Analyſis*, in a Subject that has bin ſo Engaging a Nature, as to have employ'd the Minds of the moſt Eminent Mathematicians, in all Ages, to the Conſideration of it. For as the exact Proportion between the *Diameter* and the *Circumference* can never be expreſs'd in Numbers; ſo the Improvements of thoſe Enquirers the more plainly appear'd, by how much the more Eaſie and Ready, they render'd the Way to find a Proportion the neareſt poſſible: But the Method of Series (as improv'd by Mr. *Newton*, and Mr. *Halley*) performs this with great Facility, when compared with the Intricate and Prolix Ways of *Archimedes*, *Vieta*, *Van Ceulen*, *Metius*, *Snellius*, *Lansbergius*, &c. Tho' ſome of them were ſaid to have (in this Caſe) ſet Bounds to Human Improvements, and to have ſere

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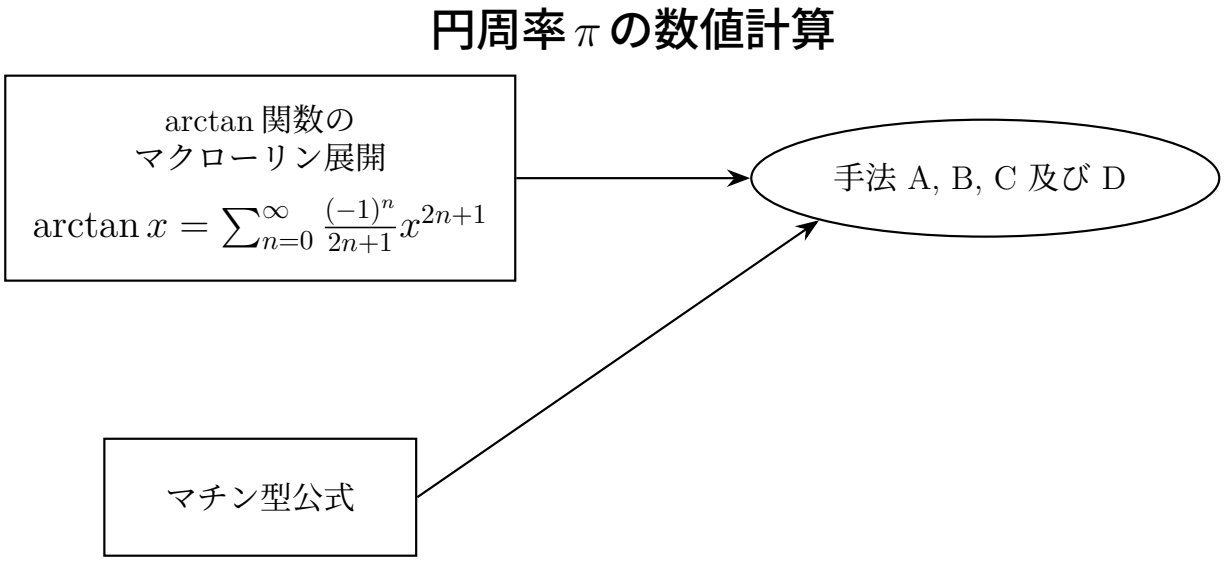
事前準備



今回は手法 A,B,C 及び D の 4 つアルゴリズムを紹介します. この 4 つの手法はすべて同じ構造で, arctan関数のマクローリン展開とマチン型公式の合わせ技になっています.

定理 1 arctan関数の微分

$$(\arctan x)' = \frac{1}{x^2+1}$$

[証明] (Fitzpatrick, 2009) の 134 ページを参考にして下さい. □
この定理より直ちに,

$$\int_0^x \frac{1}{1+t^2}\,\mathrm{d}t = \arctan x$$

が得られる.

定理 2 arctan関数のマクローリン展開
$-1 < x < 1$のとき

$$\begin{aligned}\arctan x &= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \\ &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 - \frac{1}{11}x^{11} + \frac{1}{13}x^{13} - \frac{1}{15}x^{15} + \ldots.\end{aligned}$$

[証明] (Hijab, 2016) の 127 ページを参考にする. $-1 < t < 1$を満たすtに対して, 初項が1公比が$-t^2$の無限等比級数を考えると,

$$\frac{1}{1+t^2} = \sum_{n=0}^{\infty} (-t^2)^n$$

が成立する. ここで$-1 < x < 1$なるxを固定し, 上式の両辺を0からxの範囲で積分する. すると

$$\begin{aligned}\int_0^x \frac{1}{1+t^2}\,\mathrm{d}t &= \int_0^x \sum_{n=0}^{\infty} \left(-t^2\right)^n \mathrm{d}t \\ &= \sum_{n=0}^{\infty} \int_0^x \left(-t^2\right)^n \mathrm{d}t \quad \because \text{無限和と積分の入れ替え} \\ &= \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{2n}\,\mathrm{d}t \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1} x^{2n+1}.\end{aligned}$$

上式の左辺は$\arctan x$なので、題意が得られる. □

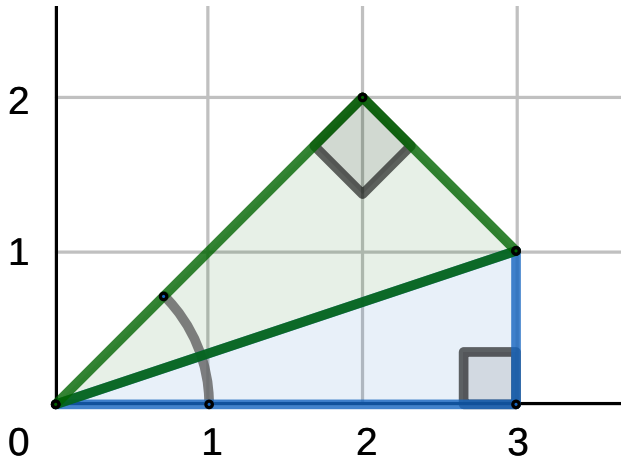
手法 A

手法 A の正当性

定理 3 マチン型公式(Euler)

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{3}$$

[証明] 下図を見て下さい.



□

系 1

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\left\{\left(\frac{1}{2}\right)^{2n+1} + \left(\frac{1}{3}\right)^{2n+1}\right\}.$$

[証明] 定理 2 及び定理 3 から直ちに得られる. □

Python による実装

系 1 に基づくアルゴリズムを実装する. その際, 数列A_n及びS_nを$n \geq 0$の範囲で

$$A_n = \sum_{k=0}^{n} \frac{(-1)^k}{2k+1}\left\{\left(\frac{1}{2}\right)^{2k+1} + \left(\frac{1}{3}\right)^{2k+1}\right\}, \quad S_n = 4A_n$$

と定義する. そして以下の python コードで数値計算を行う.

```
# Euler.py
# calculate the number pi
# [0] init
from decimal import *
getcontext().prec = 100
N = 50
digits = 50

# [1] calculate sequences A[n], S[n]
A = {}
S = {}
A[0] = (Decimal(1)/Decimal(2)) + (Decimal(1)/Decimal(3))
S[0] = 4 * A[0]

for n in range(1, N+1):
    A[n] = A[n-1] + (-1)**n / Decimal(2*n+1) * ((Decimal(1)/Decimal(2))**(2*n+1) +
(Decimal(1)/Decimal(3))**(2*n+1))
    S[n] = 4 * A[n]

# [2] print S[n]
print("n, S[n]")
for n in range(1, N+1):
    S_n = str(S[n])[:2 + digits]
    print(f"{n:0>2d}" + ", " + S_n)
```

出力を見ると, 確かに数列S_nが$\pi = 3.14159265...$に収束していることが確認できる.

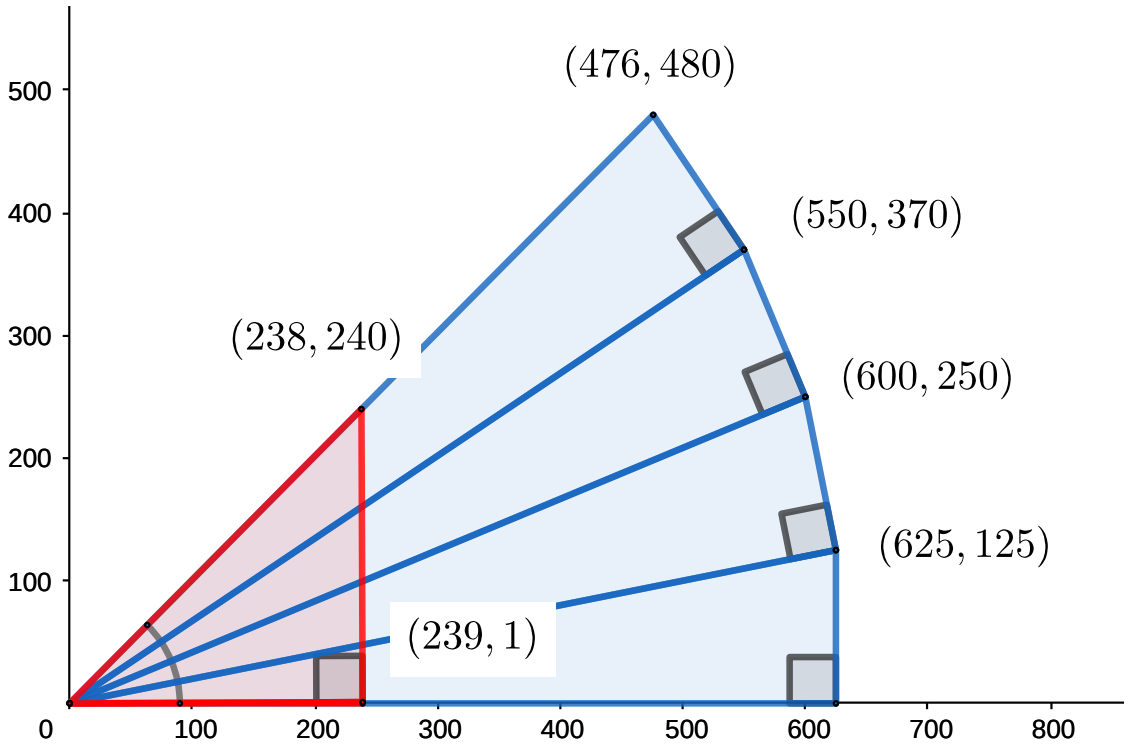
手法 B

手法 B の正当性

定理 4 マチン型公式(Machin)

$$\frac{\pi}{4} = 4\arctan\frac{1}{5} - \arctan\frac{1}{239}$$

[証明] 下図を見て下さい.



□

系 2

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left\{4\left(\frac{1}{5}\right)^{2n+1} - \left(\frac{1}{239}\right)^{2n+1}\right\}.$$

[証明] 定理 2 及び定理 4 から直ちに得られる. □

Python による実装

系 2 に基づくアルゴリズムを実装する. その際, 数列A_n及びS_nを$n \geq 0$の範囲で

$$A_n = \sum_{k=0}^{n}\frac{(-1)^k}{2k+1}\left\{4\left(\frac{1}{5}\right)^{2k+1} - \left(\frac{1}{239}\right)^{2k+1}\right\}, \quad S_n = 4A_n$$

と定義する. そして以下の python コードで数値計算を行う.

```
# Machin.py
# calculate the number pi
# [0] init
from decimal import *
getcontext().prec = 100
N = 50
digits = 50

# [1] calculate sequences A[n], S[n]
A = {}
S = {}
A[0] = 4 * (Decimal(1)/Decimal(5)) - (Decimal(1)/Decimal(239))
S[0] = 4 * A[0]
```

```
for n in range(1, N+1):
    A[n] = A[n-1] + (-1)**n / Decimal(2*n+1) * (4 * (Decimal(1)/
Decimal(5))**(2*n+1) - (Decimal(1)/Decimal(239))**(2*n+1))
    S[n] = 4 * A[n]

# [2] print S[n]
print("n, S[n]")
for n in range(1, N+1):
    S_n = str(S[n])[:2 + digits]
    print(f"{n:0>2d}" + ", " + S_n)
```

出力を見ると, 確かに数列S_nが$\pi = 3.14159265...$に収束していることが確認できる. (Arndt et al., 2012) の 13 ページによると, 1706 年マチンはこの手法 B を用いて 100 桁まで計算し世界記録を樹立しました.

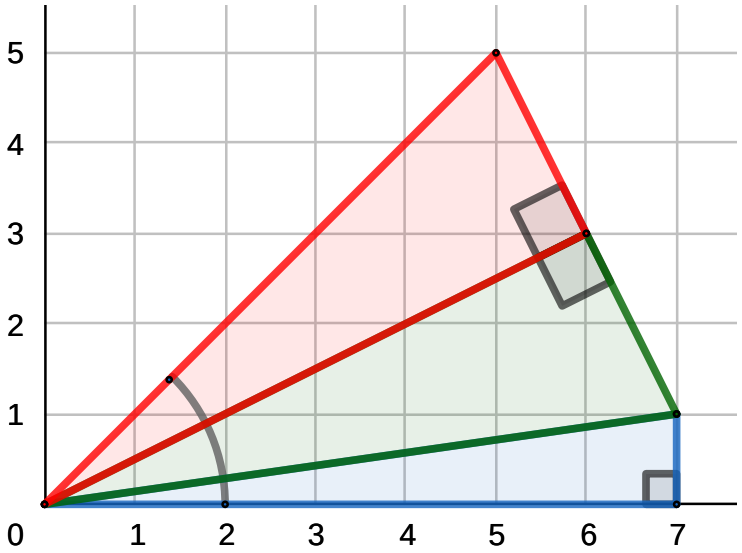
手法 C

手法 C の正当性

定理 5 マチン型公式(Hutton)

$$\frac{\pi}{4} = 2\arctan\frac{1}{3} + \arctan\frac{1}{7}$$

[証明] (Nelsen, 2015) の 115 ページを参考にする. 下図を見て下さい.



□

系 3

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left\{2\left(\frac{1}{3}\right)^{2n+1} + \left(\frac{1}{7}\right)^{2n+1}\right\}.$$

[証明] 定理 2 及び定理 5 から直ちに得られる. □

Python による実装

系 3 に基づくアルゴリズムを実装する. その際, 数列A_n及びS_nを$n \geq 0$の範囲で

$$A_n = \sum_{k=0}^{n} \frac{(-1)^k}{2k+1} \left\{ 2\left(\frac{1}{3}\right)^{2k+1} + \left(\frac{1}{7}\right)^{2k+1} \right\}, \quad S_n = 4A_n$$

と定義する. そして以下の python コードで数値計算を行う.

```
# Hutton.py
# calculate the number pi
# [0] init
from decimal import *
getcontext().prec = 100
N = 50
digits = 50

# [1] calculate sequences A[n], S[n]
A = {}
S = {}
A[0] = 2 * (Decimal(1)/Decimal(3)) + (Decimal(1)/Decimal(7))
S[0] = 4 * A[0]

for n in range(1, N+1):
    A[n] = A[n-1] + (-1)**n / Decimal(2*n+1) * (2 * (Decimal(1)/
Decimal(3))**(2*n+1) + (Decimal(1)/Decimal(7))**(2*n+1))
    S[n] = 4 * A[n]

# [2] print S[n]
print("n, S[n]")
for n in range(1, N+1):
    S_n = str(S[n])[:2 + digits]
    print(f"{n:0>2d}" + ", " + S_n)
```

出力を見ると, 確かに数列S_nが$\pi = 3.14159265...$に収束していることが確認できる.
(Nelsen, 2015) の 116 ページによると, 1789 年 Vega はこの手法 C で 143 桁まで計算しました. 但し正解していたのは 126 桁まででした.

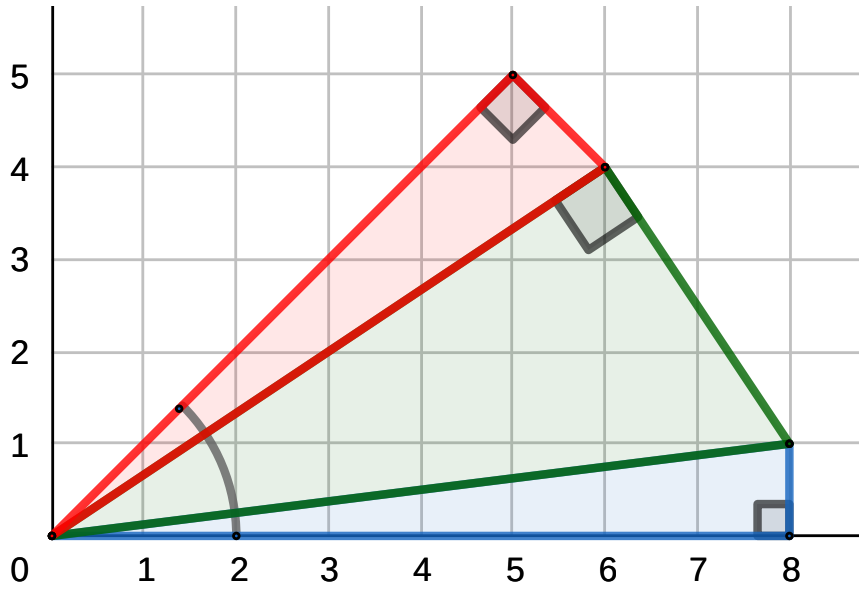
手法 D

手法 D の正当性

定理 6 マチン型公式(Strassnitzky)

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{5} + \arctan\frac{1}{8}$$

[証明] 下図を見て下さい.



□

系 4

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left\{\left(\frac{1}{2}\right)^{2n+1} + \left(\frac{1}{5}\right)^{2n+1} + \left(\frac{1}{8}\right)^{2n+1}\right\}.$$

[証明] 定理 2 及び定理 6 から直ちに得られる. □

Python による実装

系 4 に基づくアルゴリズムを実装する. その際, 数列A_n及びS_nを$n \geq 0$の範囲で

$$A_n = \sum_{k=0}^{n}\frac{(-1)^k}{2k+1}\left\{\left(\frac{1}{2}\right)^{2k+1} + \left(\frac{1}{5}\right)^{2k+1} + \left(\frac{1}{8}\right)^{2k+1}\right\}, \quad S_n = 4A_n$$

と定義する. そして以下の python コードで数値計算を行う.

```
# Strassnitzky.py
# calculate the number pi
# [0] init
from decimal import *
getcontext().prec = 100
N = 50
digits = 50

# [1] calculate sequences A[n], S[n]
A = {}
S = {}
A[0] = Decimal(1)/Decimal(2) + Decimal(1)/Decimal(5) + Decimal(1)/Decimal(8)
S[0] = 4 * A[0]

for n in range(1, N+1):
    A[n] = A[n-1] + (-1)**n / Decimal(2*n+1) * ((Decimal(1)/Decimal(2))**(2*n+1) +
(Decimal(1)/Decimal(5))**(2*n+1) + (Decimal(1)/Decimal(8))**(2*n+1))
    S[n] = 4 * A[n]
```

```
# [2] print S[n]
print("n, S[n]")
for n in range(1, N+1):
    S_n = str(S[n])[:2 + digits]
    print(f"{n:0>2d}" + ", " + S_n)
```

出力を見ると, 確かに数列S_nが$\pi = 3.14159265...$に収束していることが確認できる.
(Nelsen, 2015) の 116 ページによると, 1844 年 Dase はこの手法 D で 200 桁まで計算しました.

参考文献

Arndt, J., Lischka, C., Haenel, C., & Lischka, D. (2012). Pi - Unleashed. Springer Berlin Heidelberg.

Fitzpatrick, P. (2009). Advanced Calculus. American Mathematical Society.

Hijab, O. (2016). Introduction to Calculus and Classical Analysis. Springer International Publishing.

Jones, W. (1706). Synopsis Palmariorum Matheseos. Jeff. Wale. http://hdl.handle.net/10107/4792546

Nelsen, R. (2015). Cameos for Calculus: Visualization in the First-Year Course. MAA, The Mathematical Association of America.

Euler.py の出力
n, S[n]
01, 3.11728395061728395061728395061728395061728395061728
02, 3.14557613168724279835390946502057613168724279835390
03, 3.14085056176105558821608204324253706969756352472401
04, 3.14174119743368905082058564759372563061025185701302
05, 3.14156158787759105927068127952139535739596705912816
06, 3.14159934096619856274973211754528965658705366405750
07, 3.14159118436090672908137907388372946481309717617542
08, 3.14159298133456687617304767293551641171904062269981
09, 3.14159257960635121096510401104147911290382987070615
10, 3.14159267045068592896219887652289026992758121614467
11, 3.14159264971678824735077862446922053902179001684897
12, 3.14159265448534866704397380579026233373660422833253
13, 3.14159265338153952123818597849385885691409247817990
14, 3.14159265363845810331242127336743316294554005386619
15, 3.14159265357837256701741192316104013472004908721780
16, 3.14159265359248353692207180917269226897927541706999
17, 3.14159265358915738258560053834523148983465805552991
18, 3.14159265358994397283803346025822708599099921965575
19, 3.14159265358975740979697666470182412695884929637183
20, 3.14159265358980177539485998261260696108589224219316
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25, 3.14159265358979323170996072047776219785825372918137
26, 3.14159265358979324008900241965561888057619600781634
27, 3.14159265358979323807041510173843772018050555364853
28, 3.14159265358979323855735502486434323942946643764577
29, 3.14159265358979323843974665360688042118948666360494
30, 3.14159265358979323846818474337750763748860948523584
31, 3.14159265358979323846130092006008388252871284022487
32, 3.14159265358979323846296892340237524796371662661141
33, 3.14159265358979323846256437035293971428622711132910
34, 3.14159265358979323846266257707146202148422152425010
35, 3.14159265358979323846263871698844076608507864832434
36, 3.14159265358979323846264451858396990593924427236845
37, 3.14159265358979323846264310686239114868433166754359
38, 3.14159265358979323846264345062576259930272477690471
39, 3.14159265358979323846264336686063727747610977792137
40, 3.14159265358979323846264338728484993310653410389362
41, 3.14159265358979323846264338230183419483527097460008
42, 3.14159265358979323846264338351827627211913651254256
43, 3.14159265358979323846264338322115679921934194686725
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45, 3.14159265358979323846264338327601375215433070601366
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47, 3.14159265358979323846264338327929384204841096533689
48, 3.14159265358979323846264338327955408482126738160476
49, 3.14159265358979323846264338327949033848549194630712
50, 3.14159265358979323846264338327950595949351612475873

Machin.py の出力
n, S[n]
01, 3.14059702932606031430453110657922889814977659917261
02, 3.14162102932503442504683251711640806970624461821343
03, 3.14159177218217729501821229111232979502653517350058
04, 3.14159268240439951724025983607357586048975799672013
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17, 3.14159265358979323846264337755347132855610817514557
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19, 3.14159265358979323846264338327123657791931261035001
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23, 3.14159265358979323846264338327950286649539472156709
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30, 3.14159265358979323846264338327950288419716940163012
31, 3.14159265358979323846264338327950288419716939928768
32, 3.14159265358979323846264338327950288419716939937849
33, 3.14159265358979323846264338327950288419716939937497
34, 3.14159265358979323846264338327950288419716939937511
35, 3.14159265358979323846264338327950288419716939937510
36, 3.14159265358979323846264338327950288419716939937510
37, 3.14159265358979323846264338327950288419716939937510
38, 3.14159265358979323846264338327950288419716939937510
39, 3.14159265358979323846264338327950288419716939937510
40, 3.14159265358979323846264338327950288419716939937510
41, 3.14159265358979323846264338327950288419716939937510
42, 3.14159265358979323846264338327950288419716939937510
43, 3.14159265358979323846264338327950288419716939937510
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46, 3.14159265358979323846264338327950288419716939937510
47, 3.14159265358979323846264338327950288419716939937510
48, 3.14159265358979323846264338327950288419716939937510
49, 3.14159265358979323846264338327950288419716939937510
50, 3.14159265358979323846264338327950288419716939937510

Hutton.py の出力
n, S[n]
01, 3.13544253680308102076809559802757081668646294496634
02, 3.14207449815761167512752500489091724225233411220731
03, 3.14155123586770645370192587010952188702769927527161
04, 3.14159640711560992223984375917148454313588077891477
05, 3.14159230145587455344437603207462312507735711347606
06, 3.14159268744395758747100557138846228796177442814850
07, 3.14159265027498441761472007968104441143627074032640
08, 3.14159265391899689971857791531854923249265206396445
09, 3.14159265355672673504420632639827611094906578519732
10, 3.14159265359314542733179858509434601678893983731907
11, 3.14159265358945077741782103299907558831574271061221
12, 3.14159265358982845274183070348093257548677287949497
13, 3.14159265358978959725583149809750254176349159865464
14, 3.14159265358979361678886570826783681403899263326047
15, 3.14159265358979319898794100905335728547071934340425
16, 3.14159265358979324259679173516430997553254617259122
17, 3.14159265358979323802824546862069387035647101359683
18, 3.14159265358979323850842300414327027868737695754458
19, 3.14159265358979323845780599897422426388436739157290
20, 3.14159265358979323846315576374818846710189762321395
21, 3.14159265358979323846258899280314316410796367195257
22, 3.14159265358979323846264916848372822096535608993940
23, 3.14159265358979323846264276681558087449123754644341
24, 3.14159265358979323846264344907953308602022382488113
25, 3.14159265358979323846264337624525496322083316053647
26, 3.14159265358979323846264338403256771848869253919937
27, 3.14159265358979323846264338319877463560142678755525
28, 3.14159265358979323846264338328816765618388218003151
29, 3.14159265358979323846264338327857179521740391756231
30, 3.14159265358979323846264338327960304439230777454898
31, 3.14159265358979323846264338327949209871387896453807
32, 3.14159265358979323846264338327950404671001745177001
33, 3.14159265358979323846264338327950275878340219195231
34, 3.14159265358979323846264338327950289773844763866372
35, 3.14159265358979323846264338327950288273391221484042
36, 3.14159265358979323846264338327950288435540691056562
37, 3.14159265358979323846264338327950288418004526199089
38, 3.14159265358979323846264338327950288419902379538642
39, 3.14159265358979323846264338327950288419696845492024
40, 3.14159265358979323846264338327950288419719118728900
41, 3.14159265358979323846264338327950288419716703558637
42, 3.14159265358979323846264338327950288419716965596718
43, 3.14159265358979323846264338327950288419716937150693
44, 3.14159265358979323846264338327950288419716940240337
45, 3.14159265358979323846264338327950288419716939904588
46, 3.14159265358979323846264338327950288419716939941091
47, 3.14159265358979323846264338327950288419716939937120
48, 3.14159265358979323846264338327950288419716939937553
49, 3.14159265358979323846264338327950288419716939937505
50, 3.14159265358979323846264338327950288419716939937511

Strassnitzky.py の出力
n, S[n]
01, 3.120062500000000000000000000000000000000000000000
02, 3.145342914062500000000000000000000000000000000000
03, 3.140871041584123883928571428571428571428571428571
04, 3.141739328006604149228050595238095238095238095238
05, 3.141561763698816759472801571800595238095238095238
06, 3.141599324047591833135431066100850646749084249084
07, 3.141591186018012787346249625032201354756896749084
08, 3.141592981169975708308416648576346335804583157632
09, 3.141592579622884077100228458280087729984015173302
10, 3.141592670449009848602213261565525909679395226651
11, 3.141592649716959477355426467496915718257110592738
12, 3.141592654485331059923551614661651989758805934351
13, 3.141592653381541341840878093685310319164163351283
14, 3.141592653638457914149336817386323910277337925067
15, 3.141592653578372586754762106898804659197560433313
16, 3.141592653592483534854997671067204019417007951327
17, 3.141592653589157382802799494243125210056632161999
18, 3.141592653589943972815143649880662079774036364966
19, 3.141592653589757409799395356852396859463110100739
20, 3.141592653589801775394603792378342981627091520063
21, 3.141592653589791199874815735073169122167881969557
22, 3.141592653589793726248987326540499791442798850968
23, 3.141592653589793121531765403050979781347687387712
24, 3.141592653589793266540486986744895269846108962654
25, 3.141592653589793231709960723994886158339876252843
26, 3.141592653589793240089002419279086463430686728437
27, 3.141592653589793238070415101778801844478367122476
28, 3.141592653589793238557355024860010853436110423581
29, 3.141592653589793238439746653607345965679369389954
30, 3.141592653589793238468184743377457557391040298212
31, 3.141592653589793238461300920060089275270358057621
32, 3.141592653589793238462968923402374666707292600414
33, 3.141592653589793238462564370352939776993110715040
34, 3.141592653589793238462662577071462014713582404605
35, 3.141592653589793238462638716988440766816707240591
36, 3.141592653589793238462644518583969905860125516773
37, 3.141592653589793238462643106862391148692893736235
38, 3.141592653589793238462643450625762599301797578899
39, 3.141592653589793238462643366860637277476210250148
40, 3.141592653589793238462643387284849331065232099366
41, 3.141592653589793238462643382301834194835272156494
42, 3.141592653589793238462643383518276272119136384246
43, 3.141592653589793238462643383221156799219341960801
44, 3.141592653589793238462643383293767456922381609227
45, 3.141592653589793238462643383276013752154330706178
46, 3.141592653589793238462643383280356728320708749666
47, 3.141592653589793238462643383279293842048410965338
48, 3.141592653589793238462643383279554084821267381604
49, 3.141592653589793238462643383279490338485491946307
50, 3.141592653589793238462643383279505959493516124758